UNIVERSITY OF
KWAZULU-NATAL

University of KwaZulu-Natal

School of Engineering
Discipline of Electrical, Electronic & Computer Engineering

UNIVERSITY EXAMINATIONS 2013
OCTOBER 2013

ENEL4TB SELECTED TOPICS IN ELECTRONIC ENGINEERING - MICROWAVES

Time allowed: 2 hours

Instructions to Candidates:

1. This paper contains 4 questions
2. This paper has 17 pages excluding the cover page
3. Answer **ANY THREE** questions.
4. All questions carry marks as indicated against them
5. Students may bring and use one A4 sheet of relevant formulae
6. Programmable calculators may be used in this examination if the memory is cleared.

The following materials are provided:

1. Vector Analysis Tables
2. Formulae sheets
3. Smith Charts
4. Linear graph papers

Examiners

Prof. Thomas Afullo (Internal)
Prof. Saurabh Sinha (External)

Question 1 (20 Marks)

a) In free space, at a distance of 2.5 km from a VHF transmitter, the electric field intensity of the propagating wave at a frequency of 150 MHz, is given by:

$$\vec{E}(z,t) = 10^2 \sin(\omega t - \beta z)\hat{x}\ V/m$$

Determine:

i) The direction of wave propagation

ii) The magnetic field intensity, $\vec{H}(z,t)$

iii) The phase constant β

iv) The Poynting vector, $\vec{S}_{av} = \frac{1}{2}\mathrm{Re}\left(\vec{E}x\vec{H}^*\right)$

v) The total power received by a yagi antenna of effective aperture A_e=5 cm^2.

(10 marks)

b) A 15-metre RG-8A coaxial cable of inner conductor diameter d=2.26 mm and outer conductor diameter D=7.24 mm is used as a feeder between an 8-GHz microwave radio and a dish antenna. The dielectric material inside the cable has a dielectric constant of 1.95. If the loss per length is given by:

$$Loss(dB/m) \cong \frac{8.686}{2\pi d Z_o}\sqrt{\frac{\pi f \mu}{\sigma}}$$

If for the copper conductor, the conductivity, σ=5.8x10^7 S/m, and the magnetic permeability μ=4πx10^{-7} H/m, determine:

i) The characteristic impedance of the coaxial cable, Z_o.

ii) The total power loss inside the coaxial cable.

iii) The cut-off frequency for the coaxial cable, thus confirm that only one TEM mode operates at 8 GHz.

(10 marks)

Question 2 (20 Marks)

a) Design a circular waveguide filled with a lossless dielectric of relative permittivity ε_r=5.0. The waveguide must operate at a single dominant mode over a bandwidth of 1.2 GHz. Determine:

i) The radius of the waveguide

ii) The cut-off frequency for the dominant mode

iii) The cut-off frequency for the next lowest mode

(8 marks)

b) The main field for the dominant TE_{mnp} mode in a rectangular cavity resonator of length *a*, breadth *b*, and length *d*, can be expressed as:

$$H_z = -j2H_{zm}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right)$$

$$E_z = 0$$

i) Determine the expression for cut-off frequency for the dominant mode

ii) If a=8 cm, b=6 cm, and d=15 cm, determine the cut-off frequency for the lowest TE and TM modes in the cavity.

(8 marks)

c) Explain the principle of operation of reverberation chambers. Explain expressions for the lowest usable frequency, and how it relates to the lowest resonant frequency.

(4 marks)

Question 3 (20 Marks)

a) A reflector antenna operating in the upper UHF (800-MHz) band has a far-field radiation intensity, $U(r,\theta,\phi)$ approximated by:

$$U(r,\theta,\phi) = 20\cos^6\theta\sin^4\phi$$

where r, θ, and ϕ are the normal spherical coordinates.

Determine, for this antenna:

i) The power density, W/m^2
ii) The total radiated power, in Watts
iii) The antenna directivity, in dB
iv) The antenna gain, if the antenna efficiency is 0.63

(10 Marks)

b) You are required to design a UHF amplifier operating at 800 MHz. Use a GaAS FET with the following parameters:

$$S_{11} = 0.65\angle -95^0 \qquad S_{21} = 5\angle 115^0$$
$$S_{12} = 0.035\angle 40^0 \qquad S_{22} = 0.80\angle -35^0$$
$$Z_o = 50\ \Omega$$

i) Determine the amplifier stability using the K–Δ test
ii) Determine the centres and radii of the stability circles.
iii) Plot the circles on the Smith chart; hence identify areas of stability.

(10 Marks)

Question 4 (20 Marks)

a) Using a low-pass filter prototype, design an equal-ripple high-pass filter with a cut-off frequency at 9 GHz, an impedance of 75 Ω, and 40 dB attenuation at 6 GHz. The ripple factor is 0.5 dB. Determine:

   i) The minimum order of the filter required
   ii) The values of the prototype filter elements
   iii) The scaled element values
   iv) The actual filter circuit.

(10 Marks)

b) An rf design engineer is required to design a 3-section Chebyshev transformer to match a 50-Ω coaxial feeder line to a 120-Ω antenna. The maximum reflection coefficient allowed, Γ_m=0.015. Determine:

   i) The reflection coefficient for each section
   ii) The characteristic impedance for each section
   iii) The fractional bandwidth of the designed transformer

(10 Marks)

*********************************END OF PAPER *****************

APPENDIX 1: VECTOR OPERATORS

VECTOR ANALYSIS

Coordinate Transformations

Rectangular to cylindrical:

	$\hat{x}$	$\hat{y}$	$\hat{z}$
$\hat{\rho}$	$\cos\phi$	$\sin\phi$	0
$\hat{\phi}$	$-\sin\phi$	$\cos\phi$	0
$\hat{z}$	0	0	1

Rectangular to spherical:

	$\hat{x}$	$\hat{y}$	$\hat{z}$
$\hat{r}$	$\sin\theta\cos\phi$	$\sin\theta\sin\phi$	$\cos\theta$
$\hat{\theta}$	$\cos\theta\cos\phi$	$\cos\theta\sin\phi$	$-\sin\theta$
$\hat{\phi}$	$-\sin\phi$	$\cos\phi$	0

Cylindrical to spherical:

	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$
$\hat{r}$	$\sin\theta$	0	$\cos\theta$
$\hat{\theta}$	$\cos\theta$	0	$-\sin\theta$
$\hat{\phi}$	0	1	0

These tables can be used to transform unit vectors as well as vector components; e.g.,

$$\hat{\rho} = \hat{x}\cos\phi + \hat{y}\sin\phi$$

$$A_\rho = A_x\cos\phi + A_y\sin\phi$$

CENTRE OF EXCELLENCE
IN RURAL TELECOMMUNICATIONS
DEPT. OF ELECTRICAL ENGINEERING
UNIVERSITY OF DURBAN-WESTVILLE
P/BAG X 54001, DURBAN, 4000
TEL : 031 - 2044008

Vector Differential Operators

Rectangular coordinates:

$$\nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$$\nabla\cdot\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\bar{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2\bar{A} = \hat{x}\nabla^2 A_x + \hat{y}\nabla^2 A_y + \hat{z}\nabla^2 A_z$$

Cylindrical coordinates:

$$\nabla f = \hat{\rho}\frac{\partial f}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial f}{\partial \phi} + \hat{z}\frac{\partial f}{\partial z}$$

$$\nabla\cdot\bar{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\bar{A} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right) + \hat{z}\frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi}\right]$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2\bar{A} = \nabla(\nabla\cdot\bar{A}) - \nabla\times\nabla\times\bar{A}$$

Spherical coordinates:

$$\nabla f = \hat{r}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial\theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial f}{\partial\phi}$$

$$\nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\bar{A} = \frac{\hat{r}}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right] + \frac{\hat{\theta}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \frac{\hat{\phi}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

$$\nabla^2\bar{A} = \nabla\nabla\cdot\bar{A} - \nabla\times\nabla\times\bar{A}$$

Useful mathematical tables

C.1 A brief list of series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots |x| < 1$$

$$(1-x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \cdots |x| < 1$$

$$(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \cdots |x| < 1$$

$$1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \cdots = \infty$$

$$1 - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{4} + \cdots = \ln(2)$$

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$$

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \frac{\pi^2}{8}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n} \qquad \text{for all } x$$

C.2 A list of trigonometric identities

$$e^{\theta} = \cosh(\theta) + \sinh(\theta) = 1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \frac{\theta^4}{4!} + \cdots$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \qquad \text{where } j = \sqrt{-1}$$

$$\cosh(\theta) = \tfrac{1}{2}[e^{\theta} + e^{-\theta}]$$

$$\sinh(\theta) = \tfrac{1}{2}[e^{\theta} - e^{-\theta}]$$
$$\cos(\theta) = \tfrac{1}{2}[e^{j\theta} + e^{-j\theta}]$$
$$\sin(\theta) = \tfrac{1}{2j}[e^{j\theta} - e^{-j\theta}]$$
$$\sin(-\alpha) = -\sin(\alpha) \qquad \sin(\alpha) = \cos(\alpha - \pi/2)$$
$$\cos(-\alpha) = \cos(\alpha) \qquad \cos(\alpha) = -\sin(\alpha - \pi/2)$$
$$\cosh(j\alpha) = \cos(\alpha)$$
$$\sinh(j\alpha) = j\sin(\alpha)$$
$$\cos(j\beta) = \cosh(\beta)$$
$$\sin(j\beta) = j\sinh(\beta)$$
$$\sinh(\alpha + \beta) = \sinh(\alpha)\cosh(\beta) + \cosh(\alpha)\sinh(\beta)$$
$$\cosh(\alpha + \beta) = \cosh(\alpha)\cosh(\beta) + \sinh(\alpha)\sinh(\beta)$$
$$\sinh(\alpha + j\beta) = \sinh(\alpha)\cos(\beta) + j\cosh(\alpha)\sin(\beta)$$
$$\cosh(\alpha + j\beta) = \cosh(\alpha)\cos(\beta) + j\sinh(\alpha)\sin(\beta)$$
$$\sin(\alpha + j\beta) = \sin(\alpha)\cosh(\beta) + j\cos(\alpha)\sinh(\beta)$$
$$\sin(\alpha - j\beta) = \sin(\alpha)\cosh(\beta) - j\cos(\alpha)\sin(\beta)$$
$$\cos(\alpha + j\beta) = \cos(\alpha)\cosh(\beta) - j\sin(\alpha)\sinh(\beta)$$
$$\cos(\alpha - j\beta) = \cos(\alpha)\cosh(\beta) + j\sin(\alpha)\sinh(\beta)$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$
$$\sin(3\alpha) = 3\sin(\alpha) - 4\sin^3(\alpha)$$
$$\begin{aligned}\cos(2\alpha) &= \cos^2(\alpha) - \sin^2(\alpha)\\ &= 2\cos^2(\alpha) - 1\\ &= 1 - 2\sin^2(\alpha)\end{aligned}$$
$$\cos(3\alpha) = 4\cos^3(\alpha) - 3\cos(\alpha)$$
$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$
$$1 + \tan^2(\alpha) = \sec^2(\alpha) \qquad 1 + \cot^2(\alpha) = \csc^2(\alpha)$$
$$\sin^2(\alpha) = \tfrac{1}{2}(1 - \cos(2\alpha))$$
$$\cos^2(\alpha) = \tfrac{1}{2}(1 + \cos(2\alpha))$$
$$\sin^3(\alpha) = \tfrac{1}{4}(3\sin(\alpha) - \sin(3\alpha))$$
$$\cos^3(\alpha) = \tfrac{1}{4}(3\cos(\alpha) + \cos(3\alpha))$$
$$2\sin(\alpha)\cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta)$$
$$2\cos(\alpha)\cos(\beta) = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$
$$2\sin(\alpha)\sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$
$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

C.3 A list of indefinite integrals

In the list of integrals that follows, C is simply a constant of integration.

Let $X = \sqrt{a^2 + x^2}$

$$\int x^{1/2}\,dx = \frac{2}{3}x^{3/2} + C$$

$$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$$

$$\int X\,dx = \frac{1}{2}x\,X + \frac{a^2}{2}\ln|x + X| + C$$

$$\int xX\,dx = \frac{1}{3}X^3 + C$$

$$\int \frac{dx}{X} = \ln[x + X] + C$$

$$\int \frac{dx}{X^3} = \frac{1}{a^2}\frac{x}{X} + C$$

$$\int \frac{dx}{X^5} = \frac{1}{a^4}\left[\frac{x}{X} - \frac{1}{3}\frac{x^3}{X^3}\right] + C$$

$$\int \frac{x\,dx}{X} = X + C$$

$$\int \frac{x\,dx}{X^3} = -\frac{1}{X} + C$$

$$\int \frac{x\,dx}{X^5} = -\frac{1}{3X^3} + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}(x/a) + C$$

$$\int \frac{dx}{(a^2 + x^2)^2} = \frac{x}{2a^2(a^2 + x^2)} + \frac{1}{2a^3}\tan^{-1}(x/a) + C$$

$$\int \frac{x\,dx}{a^2 + x^2} = \frac{1}{2}\ln|a^2 + x^2| + C$$

$$\int \frac{x\,dx}{(a^2 + x^2)^2} = -\frac{1}{2(a^2 + x^2)} + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln|(a + x)/(a - x)| + C = \frac{1}{a}\tanh^{-1}(x/a) + C$$

$$\int \frac{x\,dx}{(a^2 - x^2)} = -\frac{1}{2}\ln|a^2 - x^2| + C$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$$

$$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a} + C$$

$$\int \cos^2(ax)\,dx = \frac{x}{2} + \frac{\sin(2ax)}{4a} + C$$

$$\int \sin(ax)\cos(bx)\,dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)}, \quad a \neq \pm b$$

$$\int \sin(ax)\sin(bx)\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}, \quad a \neq \pm b$$

$$\int \cos(ax)\cos(bx)\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}, \quad a \neq \pm b$$

$$\int \sin(ax)\cos(ax)\,dx = -\frac{\cos(2ax)}{4a} + C$$

$$\int \sin^n(ax)\cos(ax)\,dx = \frac{\sin^{n+1}(ax)}{(n+1)a} + C, \quad n \neq -1$$

$$\int \tan(ax)\,dx = -\frac{1}{a}\ln|\cos(ax)| + C$$

$$\int \cot(ax)\,dx = \frac{1}{a}\ln|\sin(ax)| + C$$

$$\int x\sin(ax)\,dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax) + C$$

$$\int x\cos(ax)\,dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax) + C$$

$$\int \tan^2(ax)\,dx = \frac{1}{a}\tan(ax) - x + C$$

$$\int \cot^2(ax)\,dx = -\frac{1}{a}\cot(ax) - x + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int b^{ax}\,dx = \frac{1}{a\ln(b)}b^{ax} + C$$

$$\int x e^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1) + C$$

$$\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$$

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}[a\sin(bx) - b\cos(bx)] + C$$

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}[a\cos(bx) + b\sin(bx)] + C$$

$$\int \ln(ax)\,dx = x\ln(ax) - x + C$$

$$\int x^n\ln(ax)\,dx = \frac{x^{n+1}}{n+1}\ln(ax) - \frac{x^{n+1}}{(n+1)^2} + C \qquad n \neq -1$$

$$\int \frac{1}{x}\ln(ax)\,dx = \frac{1}{2}[\ln(ax)]^2 + C$$

$$\int \sinh(ax)\,dx = \frac{1}{a}\cosh(ax) + C$$

$$\int \cosh(ax)\,dx = \frac{1}{a}\sinh(ax) + C$$
$$\int \tanh(ax)\,dx = \frac{1}{a}\ln[\cosh(ax)] + C$$
$$\int \coth(ax)\,dx = \frac{1}{a}\ln|\sinh(ax)| + C$$
$$\int \mathrm{sech}(ax)\,dx = \frac{1}{a}\sin^{-1}[\tanh(ax)] + C$$
$$\int \mathrm{csch}(ax)\,dx = \frac{1}{a}\ln|\tanh(ax/2)| + C$$
$$\int \sinh^2(ax)\,dx = \frac{\sinh(2ax)}{4a} - \frac{x}{2} + C$$
$$\int \cosh^2(ax)\,dx = \frac{\sinh(2ax)}{4a} + \frac{x}{2} + C$$
$$\int \tanh^2(ax) = x - \frac{1}{a}\tanh(ax) + C$$
$$\int \coth^2(ax)\,dx = x - \frac{1}{a}\coth(ax) + C$$
$$\int \mathrm{sech}^2(ax)\,dx = \frac{1}{a}\tanh(ax) + C$$
$$\int \mathrm{csch}^2(ax)\,dx = -\frac{1}{a}\coth(ax) + C$$

C.4 A partial list of definite integrals

$$\int_0^\infty e^{-ax}\,dx = \frac{1}{a} \qquad (a > 0)$$
$$\int_0^\infty xe^{-ax}\,dx = \frac{1}{a^2} \qquad (a > 0)$$
$$\int_0^\infty x^2e^{-ax}\,dx = \frac{2}{a^3} \qquad (a > 0)$$
$$\int_0^\infty x^ne^{-ax}\,dx = \frac{n!}{a^{n+1}} \qquad (a > 0, n > -1)$$
$$\int_0^\infty x^{1/2}e^{-ax}\,dx = \frac{1}{2a}\sqrt{\pi/a} \qquad (a > 0)$$
$$\int_0^\infty x^{-1/2}e^{-ax}\,dx = \sqrt{\pi/a} \qquad (a > 0)$$
$$\int_0^\infty e^{-ax}\sin(bx)\,dx = \frac{b}{a^2+b^2} \qquad (a > 0)$$
$$\int_0^\infty e^{-ax}\cos(bx)\,dx = \frac{a}{a^2+b^2} \qquad (a > 0)$$
$$\int_0^\infty xe^{-ax}\sin(bx)\,dx = \frac{2ab}{(a^2+b^2)^2} \qquad (a > 0)$$

$$\int_0^{\infty} xe^{-ax}\cos(bx)\,dx = \frac{a^2-b^2}{(a^2+b^2)^2} \qquad (a>0)$$

$$\int_0^{2\pi} \sin(ax)\,dx = 0 \qquad (a=1,2,3,\ldots)$$

$$\int_0^{2\pi} \cos(ax)\,dx = 0 \qquad (a=1,2,3,\ldots)$$

$$\int_0^{2\pi} \sin^2(ax)\,dx = \pi \qquad (a=1,2,3,\ldots)$$

$$\int_0^{2\pi} \cos^2(ax)\,dx = \pi \qquad (a=1,2,3,\ldots)$$

$$\int_0^{\pi} \cos(ax)\,dx = 0 \qquad (a=1,2,3,\ldots)$$

$$\int_0^{\pi} \sin(ax)\,dx = \frac{1}{a}[1-\cos(a\pi)] \qquad (a=1,2,3,\ldots)$$

$$\int_0^{\pi} \sin^2(ax)\,dx = \frac{\pi}{2} \qquad (a=1,2,3,\ldots)$$

$$\int_0^{\pi} \cos^2(ax)\,dx = \frac{\pi}{2} \qquad (a=1,2,3,\ldots)$$

$$\int_0^{\pi} \sin(ax)\sin(bx) = 0 \qquad a\neq b\ (a \text{ and } b \text{ are integers})$$

$$\int_0^{\pi} \cos(ax)\cos(bx) = 0 \qquad a\neq b\ (a \text{ and } b \text{ are integers})$$

$$\begin{aligned}\int_0^{\pi} \sin(ax)\cos(bx) &= 0 \qquad a = b\ (a \text{ and } b \text{ are integers})\\ &= 0 \qquad a\neq b \text{ but } (a+b) \text{ even}\\ &= \frac{2a}{a^2-b^2} \qquad a\neq b \text{ but } (a+b) \text{ odd}\end{aligned}$$

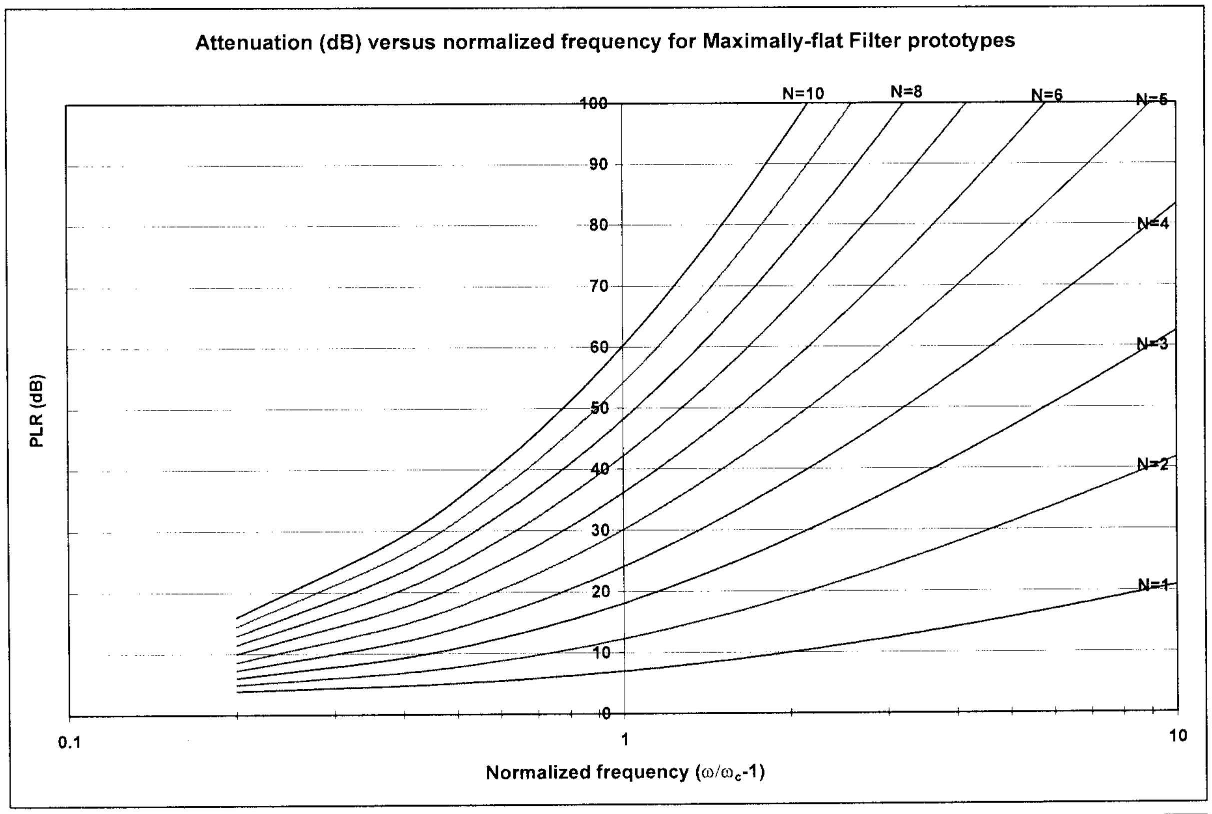
$$\int_0^{\pi/2} \sin(ax)\,dx = \frac{1}{a}[1-\cos(a\pi/2)]$$

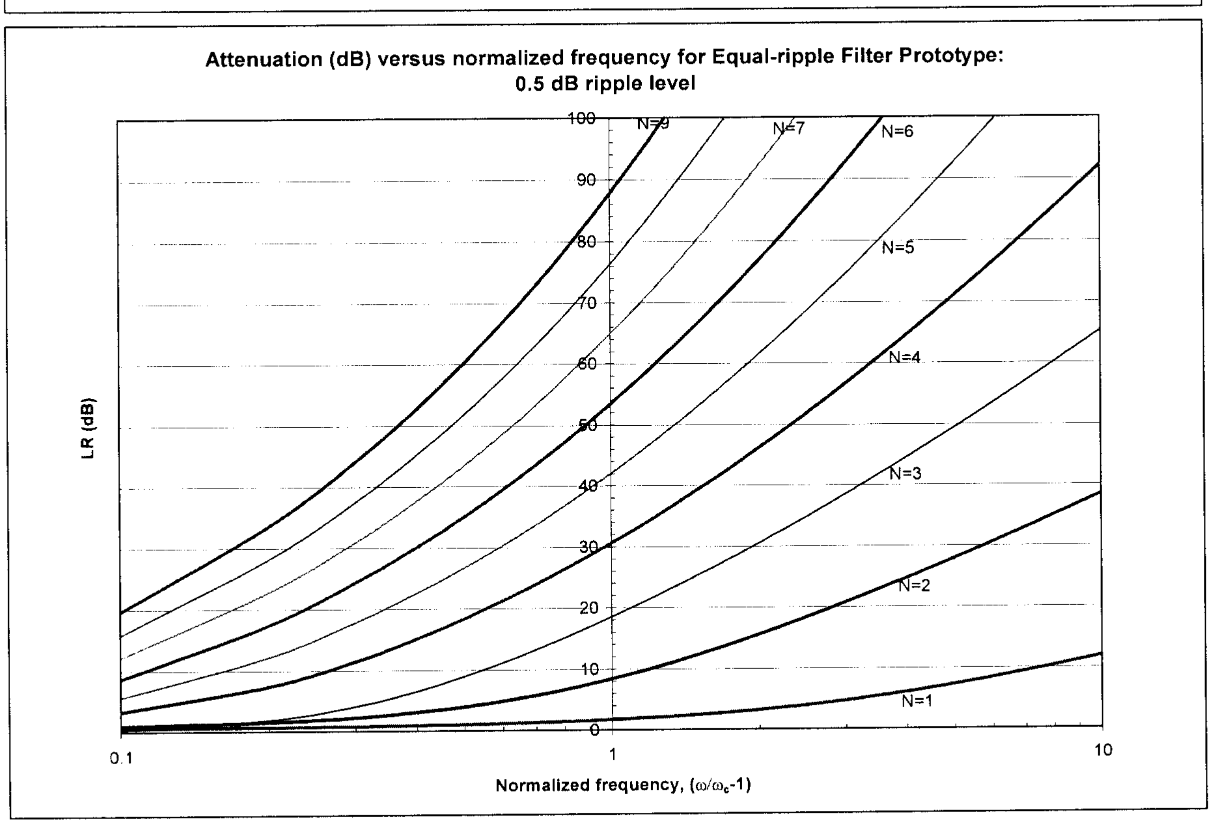
$$\int_0^{\pi/2} \cos(ax)\,dx = \frac{1}{a}\sin(a\pi/2)$$

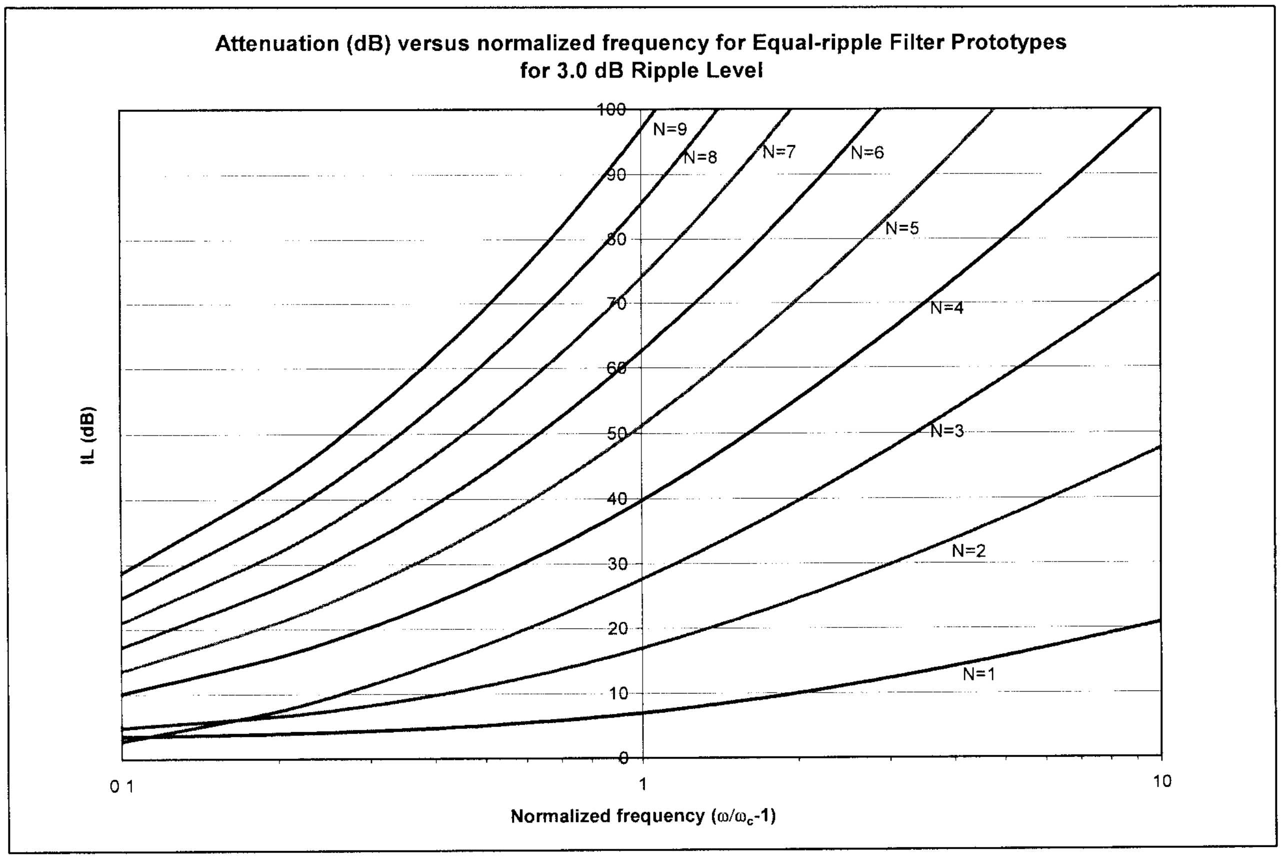
$$\int_0^{\pi/2} \sin^2(ax)\,dx = \frac{\pi}{4} \qquad (a=1,2,3,\ldots)$$

$$\int_0^{\pi/2} \cos^2(ax)\,dx = \frac{\pi}{4} \qquad (a=1,2,3,\ldots)$$

APPENDIX 2: MAXIMALLY FLAT & EQUAL-RIPPLE FILTER PLOTS AND TABLE







n	$n-th\ order\ Butterworth\ Polynomial$
1	$(s+1)$
2	$(s^2+\sqrt{2}s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765s+1)(s^2+1.848s+1)$
5	$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$
6	$(s^2+0.518s+1)(s^2+\sqrt{2}s+1)(s^2+1.932s+1)$
7	$(s+1)(s^2+0.445s+1)(s^2+1.247s+1)(s^2+1.802s+1)$
8	$(s^2+0.390s+1)(s^2+1.111s+1)(s^2+1.6663s+1)(s^2+1.962s+1)$

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	**2.0000**	**1.0000**									
2	**1.4142**	**1.4142**	**1.0000**								
3	**1.0000**	**2.0000**	**1.0000**	**1.0000**							
4	**0.7654**	**1.8478**	**1.8478**	**0.7654**	**1.0000**						
5	**0.6180**	**1.6180**	**2.0000**	**1.6180**	**0.6180**	**1.0000**					
6	**0.5176**	**1.4142**	**1.9318**	**1.9318**	**1.4142**	**0.5176**	**1.0000**				
7	**0.4450**	**1.2470**	**1.8019**	**2.0000**	**1.8019**	**1.2470**	**0.4450**	**1.0000**			
8	**0.3902**	**1.1111**	**1.6629**	**1.9615**	**1.9615**	**1.6629**	**1.1111**	**0.3902**	**1.0000**		
9	**0.3473**	**1.0000**	**1.5321**	**1.8794**	**2.0000**	**1.8794**	**1.5321**	**1.0000**	**0.3473**	**1.0000**	
10	**0.3129**	**0.9080**	**1.4142**	**1.7820**	**1.9754**	**1.9754**	**1.7820**	**1.4142**	**0.9080**	**0.3129**	**1.0000**

ELEMENT VALUES FOR MAXIMALLY FLAT LOW-PASS FILTER PROTOTYPES ($g_o=1$, $\omega_c=1$, N=1 to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	**0.6986**	**1.0000**									
2	**1.4029**	**0.7071**	**1.9841**								
3	**1.5963**	**1.0967**	**1.5963**	**1.0000**							
4	**1.6703**	**1.1926**	**2.3661**	**0.8419**	**1.9841**						
5	**1.7058**	**1.2296**	**2.5408**	**1.2296**	**1.7058**	**1.0000**					
6	**1.7254**	**1.2479**	**2.6064**	**1.3137**	**2.4758**	**0.8696**	**1.9841**				
7	**1.7372**	**1.2583**	**2.6381**	**1.3444**	**2.6381**	**1.2583**	**1.7372**	**1.0000**			
8	**1.7451**	**1.2647**	**2.6564**	**1.3590**	**2.6964**	**1.3389**	**2.5093**	**0.8796**	**1.9841**		
9	**1.7504**	**1.2690**	**2.6678**	**1.3673**	**2.7239**	**1.3673**	**2.6678**	**1.2690**	**1.7504**	**1.0000**	
10	**1.7543**	**1.2721**	**2.6754**	**1.3725**	**2.7392**	**1.3806**	**2.7231**	**1.3485**	**2.5239**	**0.8842**	**1.9841**

ELEMENT VALUES FOR EQUAL-RIPPLE LOW-PASS FILTER PROTOTYPES ($g_o=1$, $\omega_c=1$, N=1 to 10) – 0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	**1.9953**	**1.0000**									
2	**3.1013**	**0.5339**	**5.8095**								
3	**3.3487**	**0.7117**	**3.3487**	**1.0000**							
4	**3.4389**	**0.7483**	**4.3471**	**0.5920**	**5.8095**						
5	**3.4817**	**0.7618**	**4.5381**	**0.7618**	**3.4817**	**1.0000**					
6	**3.5045**	**0.7685**	**4.6061**	**0.7929**	**4.4641**	**0.6033**	**5.8095**				
7	**3.5182**	**0.7723**	**4.6386**	**0.8039**	**4.6386**	**0.7723**	**3.5182**	**1.0000**			
8	**3.5277**	**0.7745**	**4.6575**	**0.8089**	**4.6990**	**0.8018**	**4.4990**	**0.6073**	**5.8095**		
9	**3.5340**	**0.7760**	**4.6692**	**0.8118**	**4.7272**	**0.8118**	**4.6692**	**0.7760**	**3.5340**	**1.0000**	
10	**3.5384**	**0.7771**	**4.6768**	**0.8136**	**4.7425**	**0.8164**	**4.7260**	**0.8051**	**4.5142**	**0.6091**	**5.8095**

ELEMENT VALUES FOR EQUAL-RIPPLE LOW-PASS FILTER PROTOTYPES ($g_o=1$, $\omega_c=1$, N=1 to 10) – 3.0 dB Ripple

APPENDIX 3: MICROWAVE AMPLIFIER DESIGN FORMULAE

i) The K-Δ Test for Stability

$$|\Delta| = |s_{11}s_{22} - s_{12}s_{21}| < 1$$

$$|K| = \left|\frac{1 - |s_{11}|^2 - |s_{22}|^2 + \Delta^2}{2|s_{21}s_{12}|}\right| > 1$$

ii) The μ Test for Stability:

$$\mu = \frac{1 - |s_{11}|^2}{|s_{22} - \Delta s_{11}^*| + |s_{12}s_{21}|} > 1$$

iii) Input and Output Reflection Coefficients

$$|\Gamma_{in}| = \left|s_{11} + \frac{s_{12}\Gamma_L s_{21}}{1 - s_{22}\Gamma_L}\right| < 1; \quad |\Gamma_{out}| = \left|s_{22} + \frac{s_{21}\Gamma_g s_{12}}{1 - s_{11}\Gamma_g}\right| < 1$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}; \quad \Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o}$$

iv) Load (output) and source (input) stability circles with centre C_L and radius R_L:

$$C_L = \frac{(s_{22} - \Delta s_{11}^*)^*}{|s_{22}|^2 - |\Delta|^2} \quad R_L = \left|\frac{s_{12}s_{21}}{|s_{22}|^2 - |\Delta|^2}\right|$$

$$C_g = \frac{(s_{11} - \Delta s_{22}^*)^*}{|s_{11}|^2 - |\Delta|^2} \quad R_g = \left|\frac{s_{12}s_{21}}{|s_{11}|^2 - |\Delta|^2}\right|$$

v) Conjugate Matching for Maximum Gain, G_T

$$\Gamma_{gM} = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \qquad \Gamma_{LM} = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \qquad B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^* \qquad C_2 = S_{22} - \Delta S_{11}^*$$

$$G_T = G_g G_o G_L \qquad G_o = |S_{21}|^2$$

$$G_g = \frac{1}{1 - |\Gamma_{gM}|^2} \qquad G_L = \frac{1 - |\Gamma_{LM}|^2}{|1 - S_{22}\Gamma_{LM}|^2}$$

APPENDIX 4: QUARTER-WAVE TRANSFORMER DESIGN FORMULAE

i) The reflection coefficient at the n-th section is:

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

ii) Defining Γ_m, as maximum reflection coefficient that can be tolerated:

$$\frac{1}{\Gamma_m^2} = 1 + \left(\frac{2\sqrt{Z_o Z_L}}{Z_L - Z_o} \sec\theta_m \right)^2$$

$$\cos\theta_m = \frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \left(\frac{2\sqrt{Z_o Z_L}}{|Z_L - Z_o|} \right)$$

iii) The fractional bandwidth is:

$$\frac{\Delta f}{f_o} = \frac{2(f_o - f_m)}{f_o} = 2 - \frac{2f_m}{f_o} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_L - Z_o|} \right]$$

iv) The first four polynomials for the Chebyshev transformer design are:

$$T_1(x) = x; \;\; T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x; \;\; T_4(x) = 8x^4 - 8x^2 + 1$$

APPENDIX 5: WAVEGUIDE PARAMETERS

$$\beta_{g,mn} = \frac{\omega}{v_p}\sqrt{1-\frac{f_{c,mn}^2}{f^2}} = \beta\sqrt{1-\frac{f_{c,mn}^2}{f^2}}; \qquad f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$\eta_{nm}^{TM} = \frac{E_x}{H_y} = -j\frac{\alpha_{g,mn}}{\omega\varepsilon}; \qquad \eta_{nm}^{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{j\omega\mu}{\alpha_{g,mn}}$$

$$v_{p,mn} = \frac{\omega}{\beta_{g,mn}} = \frac{v_p}{\sqrt{1-\left(\frac{f_{c,mn}}{f}\right)^2}}; \qquad v_{g,mn} = \frac{d\omega}{d\beta_{g,mn}} = v_p\sqrt{1-\left(\frac{f_{c,mn}}{f}\right)^2}$$

$$P_{av} = \frac{\beta_{mn}^2 a^3 b^3}{8\left(n^2a^2+m^2b^2\right)}\frac{E_{zm}^2}{\eta_{mn}^{TM}} \qquad P_{av} = \eta_{mn}^{TE}\left[\frac{\beta_{mn}^2 a^3 b^3}{8\pi^2\left(n^2a^2+m^2b^2\right)}H_{zm}^2\right]$$

LOSSES IN A RECTANGULAR WAVEGUIDE

$$\alpha_{10} = \frac{1}{\sigma_c\delta_c\eta b}\left[\frac{1+\frac{2b}{a}\left(\frac{f}{f_{c,10}}\right)^2}{\sqrt{1-\left(\frac{f_{c,10}}{f}\right)^2}}\right]$$

BESSEL FUNCTIONS $J_{mn}(k_c a)=0$:

$n\rightarrow$ / $m\downarrow$	0	1	2	3	4
1	2.4	3.83	5.14	6.38	7.59
2	5.52	7.02	8.42	9.76	11.06
3	8.65	10.17	11.62	13.02	14.37

BESSEL FUNCTIONS $J'_{mn}(k_c a)=0$:

$n\rightarrow$ / $m\downarrow$	0	1	2	3	4
1	3.83	1.84	3.05	4.20	5.32
2	7.02	5.33	6.71	8.02	9.28
3	10.17	8.54	9.97	11.35	12.68