UNIVERSITY OF KWAZULU-NATAL
SCHOOL OF ENGINEERING
SYSTEMS AND SIMULATION - ENEL3SS
Examinations May 2014

Time: 2 hours
Total marks: 75

Examiners: Prof. S. H. Mneney
Dr. F. Ghayoor
Prof. H Xu

Instructions to candidates:

1) Your answer to Section A and B must be submitted in separate examination books.
2) Attempt ALL questions. Questions do not carry equal marks.
3) Calculators may be used. Programmable calculators must be cleared before the start of the exam.
4) The following are attached: Laplace and z-transform tables.

Modified Euler

$$\boldsymbol{x}_{i+1} = \boldsymbol{x}_i + \frac{\Delta t_{i+1}}{2}\left(\boldsymbol{f}(\boldsymbol{x}_i, t_i) + \boldsymbol{f}\left(\boldsymbol{x}_{i+1}^{EE}, t_{i+1}\right)\right)$$

Observer form

$$\dot{\boldsymbol{x}}_{obs} = \begin{bmatrix} 0 & & & -a_0 \\ 1 & 0 & & -a_1 \\ & \ddots & & \\ & 0 & \ddots\ 0 & -a_{n-2} \\ & & 1 & -a_{n-1} \end{bmatrix} \boldsymbol{x}_{obs} + \begin{bmatrix} b_0 - a_0 b_n \\ b_1 - a_1 b_n \\ \vdots \\ b_{n-2} - a_{n-2} b_n \\ b_{n-1} - a_{n-1} b_n \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & \cdots & \cdots & 0 & 1 \end{bmatrix} \boldsymbol{x}_{obs} + b_n u$$

State space

$$\boldsymbol{S} = \left[\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{B} \\ \hline \boldsymbol{C} & \boldsymbol{D} \end{array}\right], \quad \boldsymbol{P}(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}$$

2nd order lowpass:

$$M_p = e^{-\pi\zeta / \sqrt{1-\zeta^2}}, \quad t_p = \frac{\pi}{\omega_d}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}, \quad t_r = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{-\sigma}\right),$$

$$\sigma = \zeta\omega_n, \quad M_m = -20\log_{10} 2\zeta\sqrt{1-\zeta^2}, \quad \omega_m = \omega_n\sqrt{1-2\zeta^2},$$

$$\left|T(j\omega_n)\right|_{dB} = -20\log_{10} 2\zeta, \quad t_{s5\%} \approx 3/\sigma, \quad t_{s2\%} \approx 4/\sigma$$

z-transform

$$Z\{y(n-1)\} = z^{-1}Z\{y(n)\} + y_{(-1)},$$

$$Z\left\{e^{-\alpha n} y(n)\right\} = Z\{y(n)\}\big|_{z = e^{\alpha} z}$$

$$y_0 = \lim_{z\to\infty} Y(z);$$

$$\lim_{N\to\infty} y_N = \lim_{z\to 1}(z-1)Y(z)$$

bi-linear transform

$$z = \frac{1 + wT/2}{1 - wT/2}, \quad w = \frac{2}{T}\frac{z-1}{z+1}$$

Section A

Question 1 (17 marks)

Consider the following furnace system, where natural gas (i.e. fuel) and oxygen, O_2, flow into the furnace with controlled mass flow-rate of $U_{Fuel}(t)$ [Kg/s] and $U_{O_2}(t)$ [Kg/s] respectively and exhaust gas leaves the furnace with mass flow-rate of $U_{Out}(t)$ [Kg/s]. The input temperature for natural gas is T_{Fuel} [°K] and the input temperature for oxygen is T_{O_2} [°K]. 85% of natural gas is methane, CH_4, which reacts (i.e. burn) with oxygen at a rate $r(t)$ [Kg CH_4 per s] forming carbon dioxide, CO_2, steam water, H_2O, and releasing energy. The reaction is

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

which requires 4 Kg of O_2 per Kg of CH_4 and produces 2.75 Kg of CO_2 and 2.25 Kg of steam water. It also releases 55.5 MJ of heat energy per Kg of CH_4 burned. 10% of this energy is expelled along with exhaust gas. The heat capacity of exhaust gas, which is a combination CO_2, H_2O, O_2 and inert gases is C_g [J/Kg/°K]. The heat capacity for natural gas and oxygen are C_{Fuel} [J/Kg/°K] and C_{O_2} [J/Kg/°K] respectively. Recall that the heat energy is $Q = m\,C\,T$.

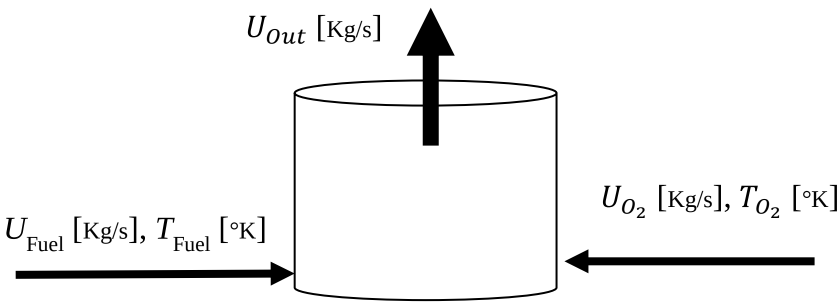

Figure Q1

a) Find the state differential equations. Clearly identify state variables, inputs and parameters. (15 marks)

b) Find the algebraic output equation for the temperature of the exhaust gas. (2 marks)

Question 2 (8 marks)

Use Modified Euler to simulate $\dot{\boldsymbol{x}} = \begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} x_2 + x_1 x_2 \\ log(x_1) + x_2 t \end{pmatrix}$ with initial condition, $\boldsymbol{x_0} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $t_0 = 1$ for one step with $\Delta t = 0.5$.

Section B

Question 3 (10 marks)

In a magnetic levitation experiment a metallic object is held up in the air suspended under an electromagnet. The vertical displacement of the object can be described by the following nonlinear equation

$$m\frac{d^2x}{dt^2}=mg-k\frac{i^2}{x^2},$$

where m is the mass of the metallic object,
g is the gravity acceleration constant,
k is a positive constant,
x is the distance between the electromagnet and the metallic object (output signal), and
i is the electromagnet's (current input signal).

a) If equilibrium is achieved when the current $i=I_0$ and the vertical displacement $x=X_0$ derive the condition for equilibrium. (2 marks)

b) Linearize the equation about the equilibrium point found in part (a) and show that the resulting transfer function obtained from the linearized differential equation can be expressed as

$$\frac{\Delta X(s)}{\Delta I(s)}=-\frac{a}{s^2-b^2}$$ where $a>0$. (8 marks)

Question 4 (8 marks)

Find the state space model of the electrical circuit of Figure Q4. In the figure, u_1 is a voltage source, x_2 and x_3 are currents while x_1 and y_1 are voltages across corresponding devices. The x_i are state space variables while y_i are output variables. (8 marks)

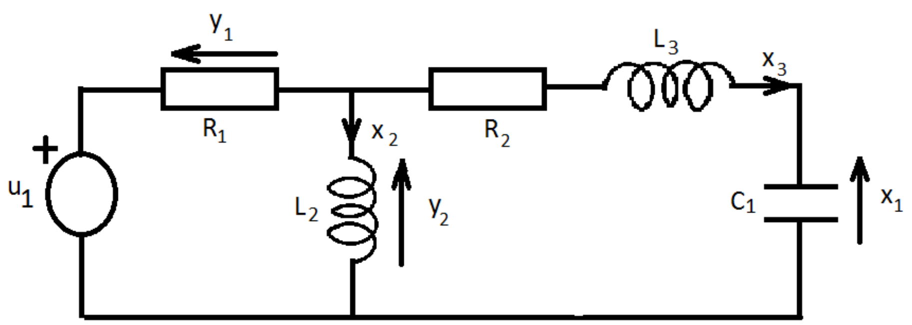

Figure Q4

Question 5 (16 marks)

a) A state space representation of a system is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0.8 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(i) Show that the system is state controllable. (4 marks)

(ii) Obtain the transfer function H(z)= Y(z)/U(z) assuming zero initial conditions. (4 marks)

b) Use the plot of the transient response of the second order system given in Figure Q5 to obtain the transfer function of the system. (8 marks)

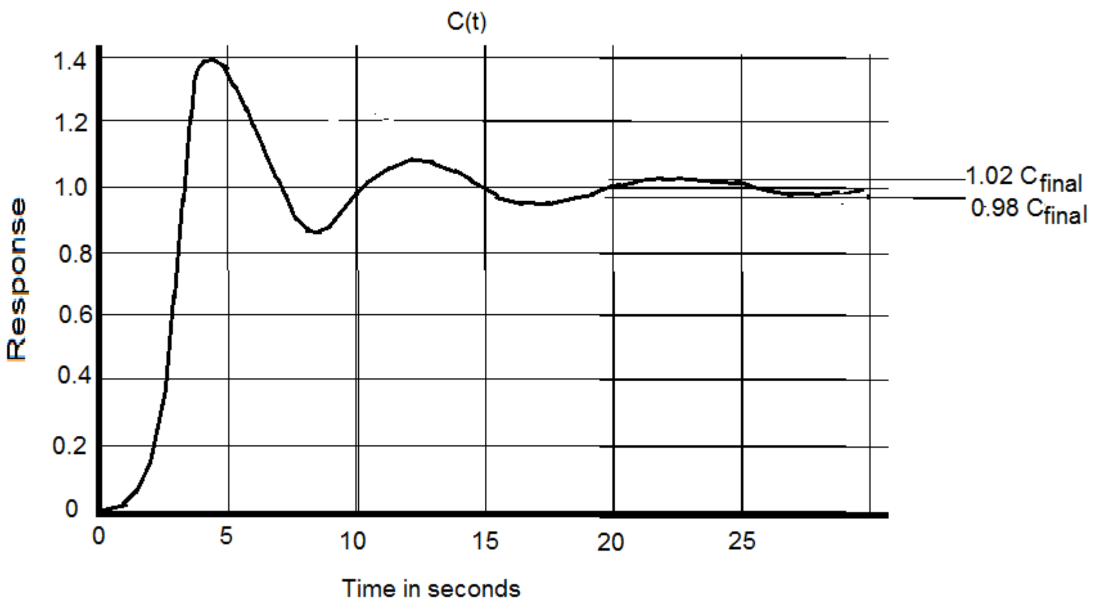

Figure Q5

Question 6 (8 marks)

A system whose input is a step signal, is given by the following state space equations

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad \mathbf{y} = \begin{bmatrix} 2 & 1 \end{bmatrix} \mathbf{x} \;, \; \mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Obtain the state transition matrix. (8 marks)

Question 7 (8 marks)

Find the transfer function, $G(s)=\frac{X_2(s)}{F(s)}$ for the translational mechanical system shown in Figure Q7. (8 marks)

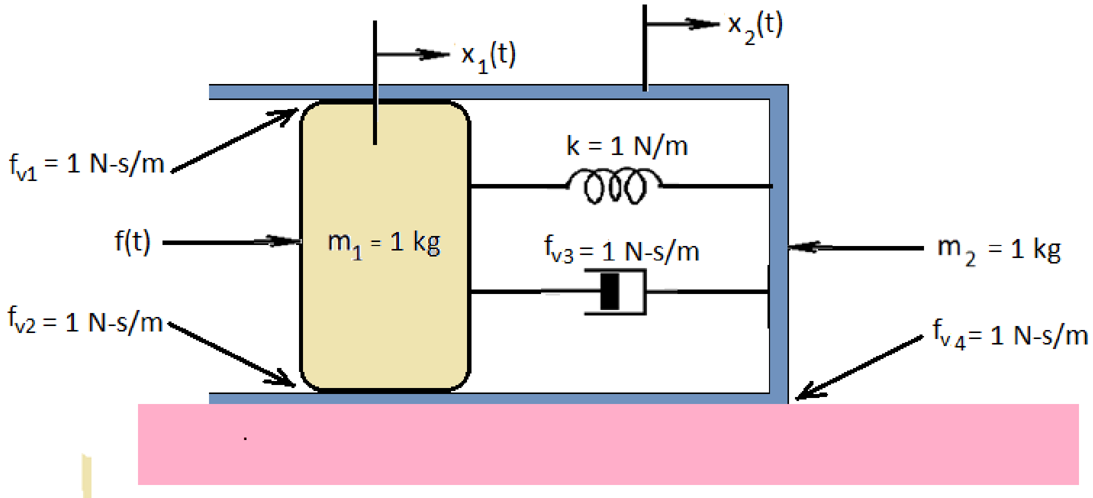

Figure Q7

RELATED LAPLACE, AND Z TRANSFORMS — TRANSIENTS AND TRANSFER FUNCTIONS

$\mathcal{Z}\{x(iT)\}$	$x(t)$	$\mathcal{L}\{x(t)\}$ / $P_s(s)$	$P_z(z) = \frac{z-1}{z}\mathcal{Z}\left\{\left[\mathcal{L}^{-1}\left\{\frac{P_s(s)}{s}\right\}\right]_{t=iT}\right\}$
not defined	$\delta(t)$	1	1
1	any $f(t)$ with $f(iT)=\begin{cases}1, i=0\\0, i\neq 0\end{cases}$	not defined	not defined
$\frac{z}{z-1}$	1	$\frac{1}{s}$	$\frac{T}{z-1}$
$\frac{Tz}{(z-1)^2}$	t	$\frac{1}{s^2}$	$\frac{T^2(z+1)}{2(z-1)^2}$
$\frac{z}{z-e^{-aT}}$	e^{-at}	$\frac{1}{s+a}$	$\frac{1-e^{-aT}}{\left[z-e^{-aT}\right]a}$
$\frac{Tze^{-aT}}{\left(z-e^{-aT}\right)^2}$	te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{\left[1-e^{-aT}(1+aT)\right]z+\left[e^{-aT}\left(e^{-aT}-1+aT\right)\right]}{\left[z-e^{-aT}\right]^2 a^2}$
$\frac{ze^{-aT}\sin(bT)/b}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$	$e^{-at}\frac{\sin(bt)}{b}$	$\frac{1}{s^2+2as+a^2+b^2}$	$\frac{\left[1-e^{-aT}\left(\cos(bT)+\frac{a}{b}\sin(bT)\right)\right]z+\left[e^{-aT}\left(e^{-aT}-\cos(bT)+\frac{a}{b}\sin(bT)\right)\right]}{\left[z^2-2ze^{-aT}\cos(bT)+e^{-2aT}\right]\left(a^2+b^2\right)}$
$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$	$e^{-at}\cos(bt)$	$\frac{s+a}{s^2+2as+a^2+b^2}$	$\frac{\left[1-e^{-aT}\left(\cos(bT)-\frac{b}{a}\sin(bT)\right)\right]z+\left[e^{-aT}\left(e^{-aT}-\cos(bT)-\frac{b}{a}\sin(bT)\right)\right]}{\left[z^2-2ze^{-aT}\cos(bT)+e^{-2aT}\right]\left(a^2+b^2\right)/a}$
$\frac{z(-1)^{m-1}}{(m-1)!}\frac{\partial^{m-1}}{\partial a^{m-1}}\frac{1}{z-e^{-at}}$ or $\frac{z(-T)^{m-1}}{(m-1)!}\left[\frac{\partial}{\partial z}z\ldots\right]^{m-1}\frac{1}{z-e^{-at}}$	$\frac{t^{m-1}}{(m-1)!}e^{-at}$, $m\geq 1$	$\frac{1}{(s+a)^m}$	$\frac{1}{a^m}\left(1-(z-1)\sum_{v=1}^{m}\frac{(-a)^{v-1}}{(v-1)!}\frac{\partial^{v-1}}{\partial a^{v-1}}\frac{1}{z-e^{-at}}\right)$ or $\frac{1}{a^m}\left(1-(z-1)\sum_{v=1}^{m}\frac{(-aT)^{v-1}}{(v-1)!}\left[\frac{\partial}{\partial z}z\ldots\right]^{v-1}\frac{1}{z-e^{-at}}\right)$
$\frac{z(-T)^{m-1}}{(m-1)!}\left[\frac{\partial}{\partial z}z\ldots\right]^{m-1}\frac{1}{z-1}$	$\frac{t^{m-1}}{(m-1)!}$, $m\geq 1$	$\frac{1}{s^m}$	$(z-1)\frac{(-T)^m}{m!}\left[\frac{\partial}{\partial z}z\ldots\right]^m\frac{1}{z-1}$

T is the time-interval of equidistant sampling of signals and holding of sequences. a can be a complex number.
The given discrete-time transfer functions are based on *stair-case inputs*: $u(t)=u_i$ for $t_i \leq t < t_i + T$.

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Dr.-Ing. habil. Eduard Eitelberg