UNIVERSITY OF KWAZULU-NATAL
SCHOOL OF ENGINEERING
CONTROL SYSTEMS 2 – ENEL4CS
Examinations May 2014

Time: 2 hours | Examiners: Dr F Ghayoor
Total marks: 70 | Prof. M Braae

Instructions to candidates:

1) Attempt ALL questions. Questions do not carry equal marks.
2) Candidates are allowed to bring 1 A4 sheet of notes into the examination.
3) Calculators may be used.
4) Laplace and z-transform tables are attached. Nichols and inverse Nichols charts are attached.

Question 1 - System identification (15 marks)

Using the simple second order system, $P = \frac{b_0}{s^2+a_1s+a_0}$, as an example, explain the method used to obtain the parameters from measurements of arbitrary (but sufficiently exciting) inputs using least square estimate. (15)

Question 2 - Template and specification (20 marks)

Given $P(s) = \frac{k}{(s/\omega_n)^2+2\xi(s/\omega_n)+1}$; $k \in [3,10], \omega_n \in [1,3], \xi = 0.7$ and a nominal plant $P_n(s) = \frac{3}{s^2+1.4s+1}$

a) Draw the plant template for $\omega = 2$ rad/s showing the nominal. (10)

b) Using your template form (2a) and the appropriate chart, find nominal bounds for the specifications,

$$\left|\frac{0.9}{(s+1)(s/2)^2+(s/2)+1}\right| \leq |T_{Y/R}| \leq \left|\frac{1.02}{(s/2)^2+0.8(s/2)+1}\right|$$

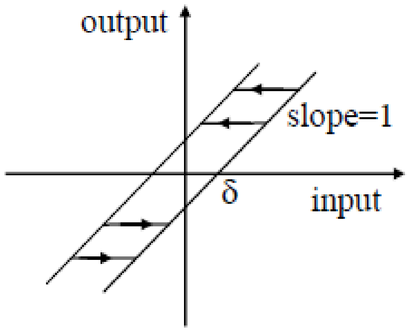
(10)

If you do not have a sensible looking answer for Question (2a), you may use a template with the following points:

	P_0			
$\lvert P\rvert_{dB}$	25	25	36	36
∠p	-105°	-27°	-27°	-105°

Question 3 - Describing function analysis (15 marks)

Friction controlled backlash has the input-output behaviour shown, with slope k=1 and dead-band 2δ.

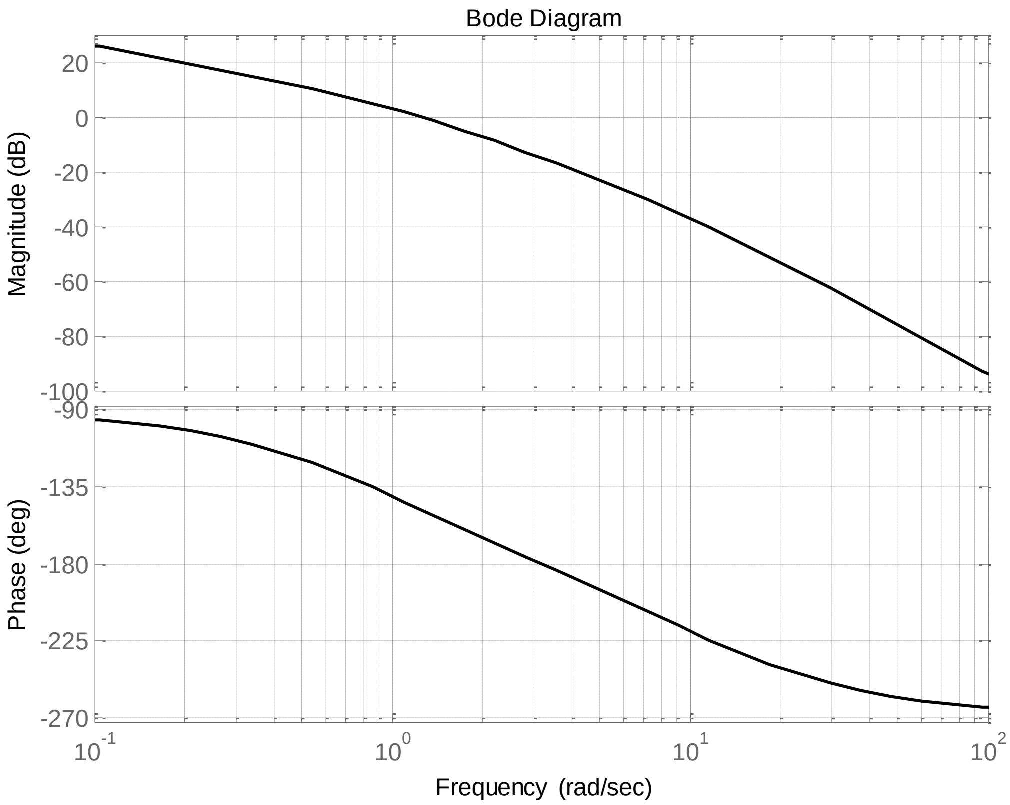


a) Sketch the output of the slack actuator for input $x(t) = X sin\omega t$. (5)

b) Given the following values of $-1/N$, analyze limit cycle possibilities for the loop transfer function,

$L = \frac{4}{s(s+1)(0.1s+1)}$ on an inverse Nichols chart. The bode plot for $L(j\omega)$ is given below. (10)

X	0.055	0.060	0.075	0.100	0.200	0.400	1.000	10.00
$dB(-1/N(X))$	30.8	20.6	7.8	3.2	1.5	0.55	0.15	0.005
$\arg(-1/N(X))$	-170	-162	-136	-115	-105	-100	-96	-91



Question 4 - Digital design (20 marks)

a) Design a digital PI controller for the continuous time plant, $P(s) = \frac{3}{s/2+1}$ for the specification below. Allow 10° for sampling effect at the gain cross-over frequency. Specify sampling rate, T, and $G(w)$. (calculation of $G(z)$is not required.) (15)

i) Zero steady state error to a step output disturbance

ii)$\left|\frac{1}{1+L}\right| \leq -20dB, \omega = 1$ rad/s

iii)$\left|\frac{1}{1+L}\right| \leq 6dB, \forall\omega$

b) Given a digital PI controller, $G(w) = \frac{K_p\left(w+\frac{1}{T_i}\right)}{w}$ and sampling time h, write down the controller algorithm (in terms of K_p, T_i and h) as a difference equation. (5)

Related Laplace, and z transforms — transients and transfer functions

$\mathcal{Z}\{x(iT)\}$	$x(t)$	$\mathcal{L}\{x(t)\} \big/ P_s(s)$	$P_z(z) = \frac{z-1}{z}\mathcal{Z}\left\{\left[\mathcal{L}^{-1}\left\{\frac{P_s(s)}{s}\right\}\right]_{t=iT}\right\}$
not defined	$\delta(t)$	1	1
1	any $f(t)$ with $f(iT) = \begin{cases}1, & i = 0\\ 0, & i \neq 0\end{cases}$	not defined	not defined
$\frac{z}{z-1}$	1	$\frac{1}{s}$	$\frac{T}{z-1}$
$\frac{Tz}{(z-1)^2}$	t	$\frac{1}{s^2}$	$\frac{T^2(z+1)}{2(z-1)^2}$
$\frac{z}{z-e^{-aT}}$	e^{-at}	$\frac{1}{s+a}$	$\frac{1-e^{-aT}}{\left[z-e^{-aT}\right]a}$
$\frac{Tze^{-aT}}{\left(z-e^{-aT}\right)^2}$	te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{\left[1-e^{-aT}(1+aT)\right]z+\left[e^{-aT}\left(e^{-aT}-1+aT\right)\right]}{\left[z-e^{-aT}\right]^2 a^2}$
$\frac{ze^{-aT}\sin(bT)/b}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$	$e^{-at}\frac{\sin(bt)}{b}$	$\frac{1}{s^2+2as+a^2+b^2}$	$\frac{\left[1-e^{-aT}\left(\cos(bT)+\frac{a}{b}\sin(bT)\right)\right]z+\left[e^{-aT}\left(e^{-aT}-\cos(bT)+\frac{a}{b}\sin(bT)\right)\right]}{\left[z^2-2ze^{-aT}\cos(bT)+e^{-2aT}\right]\left(a^2+b^2\right)}$
$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$	$e^{-at}\cos(bt)$	$\frac{s+a}{s^2+2as+a^2+b^2}$	$\frac{\left[1-e^{-aT}\left(\cos(bT)-\frac{b}{a}\sin(bT)\right)\right]z+\left[e^{-aT}\left(e^{-aT}-\cos(bT)-\frac{b}{a}\sin(bT)\right)\right]}{\left[z^2-2ze^{-aT}\cos(bT)+e^{-2aT}\right]\left(a^2+b^2\right)/a}$
$\frac{z(-1)^{m-1}}{(m-1)!}\frac{\partial^{m-1}}{\partial a^{m-1}}\frac{1}{z-e^{-at}}$ or $\frac{z(-T)^{m-1}}{(m-1)!}\left[\frac{\partial}{\partial z}z\ldots\right]^{m-1}\frac{1}{z-e^{-at}}$	$\frac{t^{m-1}}{(m-1)!}e^{-at},\ m \geq 1$	$\frac{1}{(s+a)^m}$	$\frac{1}{a^m}\left(1-(z-1)\sum_{\nu=1}^{m}\frac{(-a)^{\nu-1}}{(\nu-1)!}\frac{\partial^{\nu-1}}{\partial a^{\nu-1}}\frac{1}{z-e^{-at}}\right)$ or $\frac{1}{a^m}\left(1-(z-1)\sum_{\nu=1}^{m}\frac{(-aT)^{\nu-1}}{(\nu-1)!}\left[\frac{\partial}{\partial z}z\ldots\right]^{\nu-1}\frac{1}{z-e^{-at}}\right)$
$\frac{z(-T)^{m-1}}{(m-1)!}\left[\frac{\partial}{\partial z}z\ldots\right]^{m-1}\frac{1}{z-1}$	$\frac{t^{m-1}}{(m-1)!},\ m \geq 1$	$\frac{1}{s^m}$	$(z-1)\frac{(-T)^m}{m!}\left[\frac{\partial}{\partial z}z\ldots\right]^m\frac{1}{z-1}$

T is the time-interval of equidistant sampling of signals and holding of sequences. a can be a complex number.
The given discrete-time transfer functions are based on *stair-case inputs*: $u(t) = u_i$ for $t_i \leq t < t_i + T$.

Printed on 28 January, 1999.

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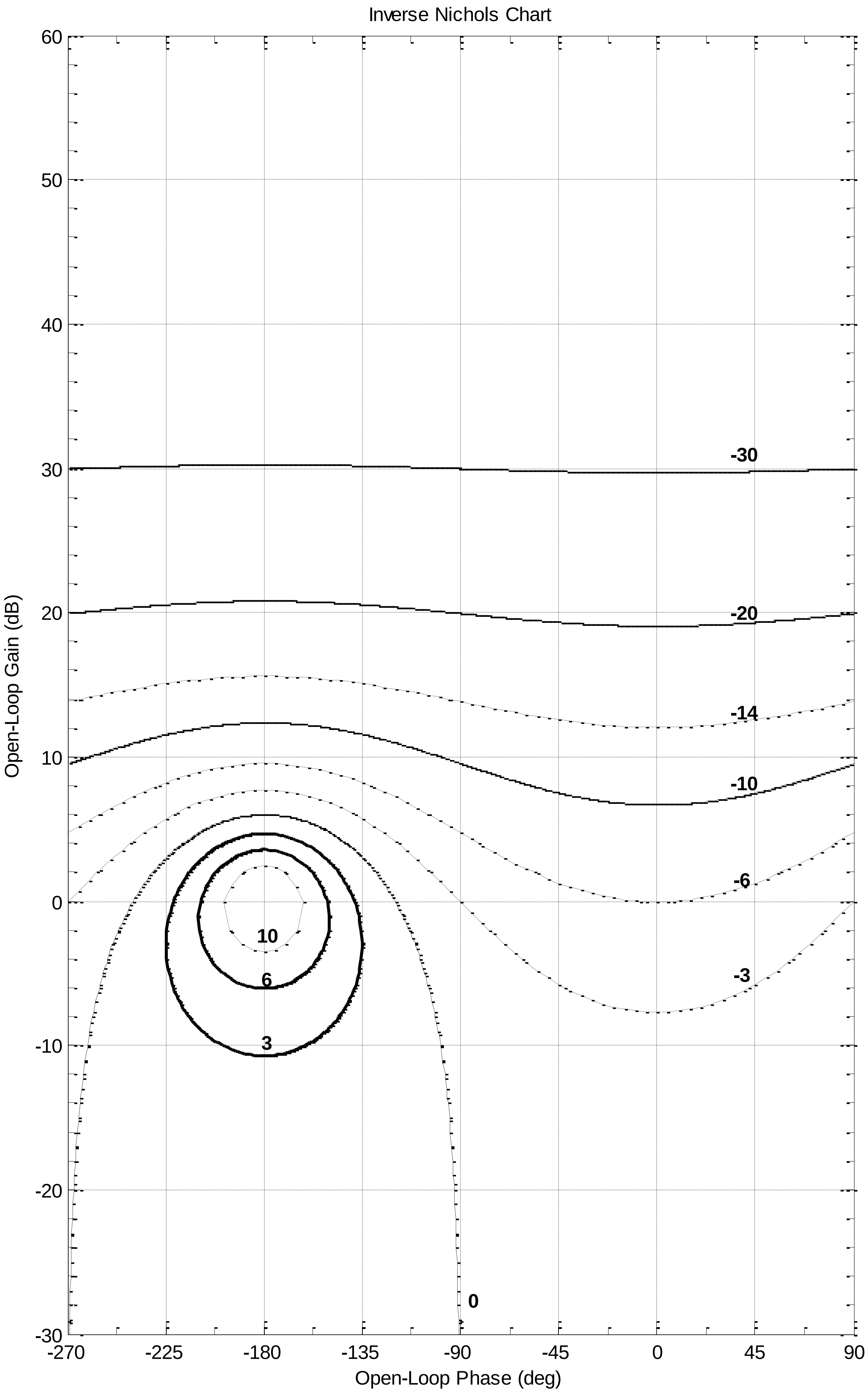
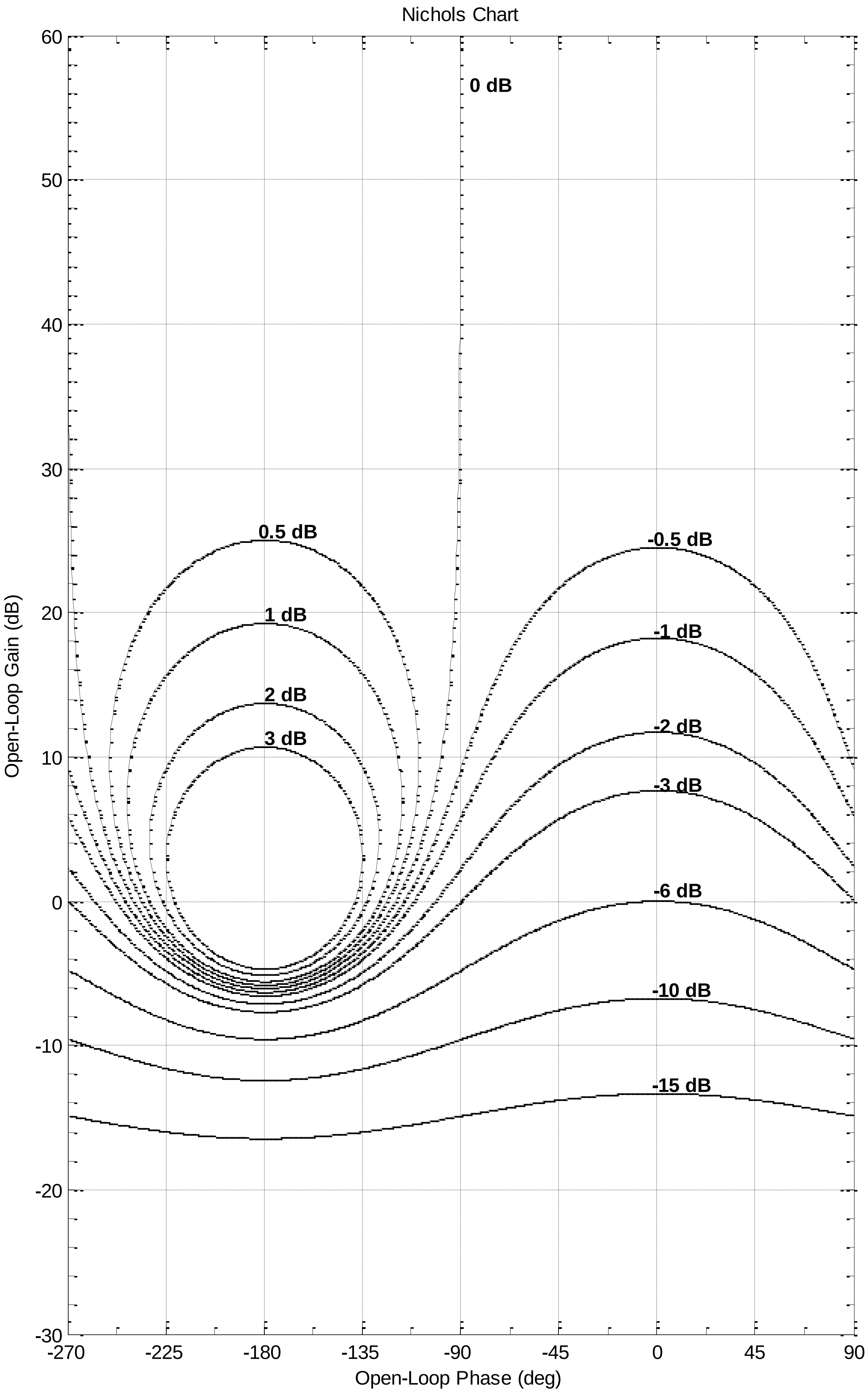

Inverse Nichols Chart
Open-Loop Gain (dB)
Open-Loop Phase (deg)
60
50
40
30
20
10
0
-10
-20
-30
-270
-225
-180
-135
-90
-45
0
45
90
-30
-20
-14
-10
-6
-3
10
6
3
0


Nichols Chart
0 dB
0.5 dB
1 dB
2 dB
3 dB
-0.5 dB
-1 dB
-2 dB
-3 dB
-6 dB
-10 dB
-15 dB
60
50
40
30
20
10
0
-10
-20
-30
-270
-225
-180
-135
-90
-45
0
45
90
Open-Loop Gain (dB)
Open-Loop Phase (deg)