University of KwaZulu-Natal

School of Engineering

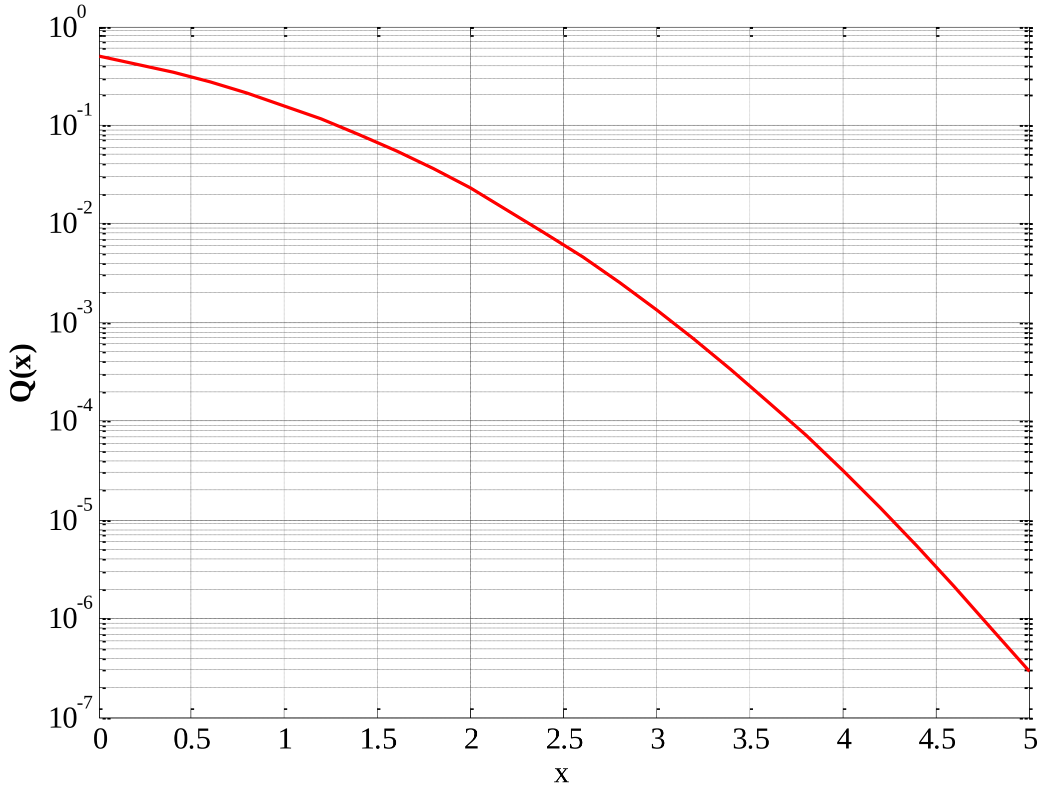
Examinations: June 2014

ENEL4DCH1 : Digital Communications

Duration: **2 Hours** Marks: 100

Internal examiner: Prof H. Xu
Internal Moderator: Prof S H Mneney
External examiner: Dr. L Cheng

Instructions : 1. Answer all questions.
2. The Gaussian Q-function is shown below. You will use it to calculate bit error rate (BER) in the exam.



Question 1 Waveform Coding [15 marks]

(1) Concepts
   (a) (**1 mark**) What is PCM?

   (b) (**1 mark**) What is DPCM?

   (c) (**1 mark**) What are the basic operations in most of waveform coding?

(2) Consider a signal v with a pdf $f(v)$

$$f(v) = \begin{cases} \dfrac{1}{24} & -4 < v \le -1 \\ K & -1 < v \le 1 \\ \dfrac{1}{24} & 1 < v \le 4 \\ 0 & elsewhere \end{cases}$$

Assume that a uniform quantizer is used.
   (a) (**2 marks**) Find K;

   (b) (**2 marks**) Determine the quantization step size and the output of the quantizer if there are 4 levels (Q=8);

   (c) (**4 marks**) Calculate the variance of the quantization error for $0 < v \le 2$.

   (d) (**4 marks**) Calculate the signal power for $0 < v \le 2$ after quantization.

Question 2 Information Theory: Source coding [15 marks]

A 100×100 source data has five symbols: A, B, C, D, and E. We would like to encode the source data in a lossless way.

(a) (**3 marks**) What is the minimum number of bits required if no additional symbol distribution is available?

(b) (**4marks**) What is the minimum number of bits required if the symbols are distributed as follows: the first one indicates the symbol and the second one indicates the number of symbols corresponding to the symbol. (A,2000); (B,4000); (C,1000); (D,1000); (E,2000).

(c) (**8marks**) For each of the above two cases, construct an optimal code assuming each symbol is coded individually.

Question 3 Information Theory: Entropy and Mutual information [15 marks]

(3.1) Let X and Y denote two discrete random variables with joint distribution $P(x, y)$.

(a) Show that

$$H(X) = -\sum_{y}\sum_{x} P(x, y)\log P(x)$$

$$H(Y) = -\sum_{y}\sum_{x} P(x, y)\log P(y)$$

(b) Use the above result to show that $H(X;Y) \leq H(X) + H(Y)$.

(3.2) Consider an information source A emitting the symbols(a_1, a_2, a_3, a_4). $p(a_1) = 0.4$, $p(a_2) = q_2$, $p(a_3) = q_3$**,** and , $p(a_4) = q_4$. Prove the entropy $H(A)$ of source A is maximized only for $p(a_2) = p(a_3) = p(a_4) = 0.2$**.**

Question 4 Channel coding: linear block coding [15 marks]

Consider a (6,3) linear block code *C* with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

(a) [**2 marks**] Determine the parity check matrix H of *C* ;

(b) [**1 mark**] How any error patterns of C ?

(c) [**1 mark**] How many errors can the code correct?

(d) [**1 mark**] What is the generator matrix for the dual code?

(e) [**4 marks**] Construct the following table;

Syndrome	Error Pattern

(f)[**4 mark**] Use pictorial representation to decode the codeword $r = [1\ 1\ 0\ 1\ 1\ 1]$;

(g)[**2 marks**] List all error patterns for syndrome being 1 0 0.

Question 5 Channel Coding: convolutional coding [15 marks]

Consider a (3,1,2) convolutional code $g_1 = [1\ 1\ 0], g_2 = [1\ 0\ 1], g_3 = [1\ 1\ 1]$.

(1) [**2 marks**] What is the rate and constraint of the convolutional code;

(2) [**4 marks**] Draw the state transition table;

Current state	input	Next state	Output

(3) [**4 marks**] A trellis of the convolutional code for 4 time intervals is shown in Fig.5.1. If a sequence (000 000 000 000) is transmitted. List all possible decoded sequences.

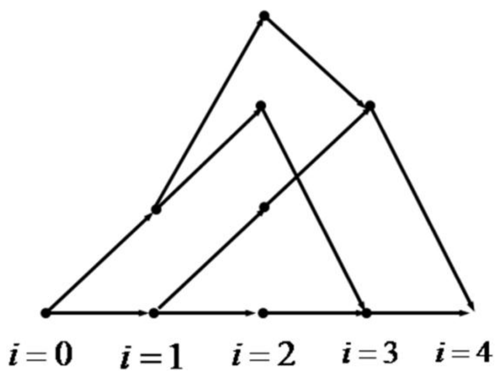


Fig. 5.1

(4) [**5 marks**] The received sequence is (000 111 011 111 011), where the leftmost bit is the first received bit. Use Viterbi algorithm to decode the received sequence (assume that the initial state is "00", and the final state is also "00").

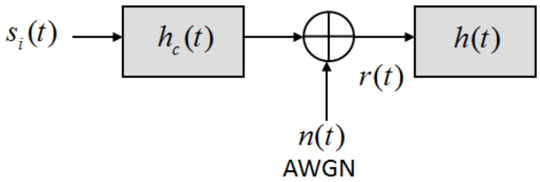
Question 6 Digital modulation [15 marks]

(1) [**5 marks**] A 8QAM signal is transmitted over an AWGN channel. At receiver, the received signal is given by $y = x + n$,

where $x \in \{1+j;\ 3+j;\ 1-j;\ 3-j;\ -1+j,;-3+j,;-1-j,-3-j\}$, , n is the AWGN with $n \sim N(0,\sigma^2)$.

Derive σ^2 for $\frac{E_b}{N_0} = 10dB$.

(2) [**5 marks**] A transmission system is given by the following diagram



Assume $h_c(t) = \delta(t)$. Prove the optimum receive filter $h(t) = h_{opt}(t) = s_i^*(T - t)$.

(3) [**5 marks**] Consider the following modulation scheme:

$$s(t) = A(t)cos(2\pi f_c t) - B(t)sin(2\pi f_c t)$$

where $A(t) \in \{-3, -1, 1, 3\}$ and $B(t) \in \{-4, -2, 2, 4\}$

(a) Write the in-phase and quadrature representation of this signal;

(b) Derive the baseband equivalent model of the signal.

Question 7 Coding and modulation (10 marks)

Design a modulation and coding scheme for a wireless communication system. The system requires a 300kbps data rate in a 200kHz channel. A matched filter receiver is to be used. QPSK and 8-PSK modulations and (n=63, k=36, t=5) linear block code, (n=63, k=24, t=7) linear block code and (n=63, k=16, t=11) linear block code are to be considered, where t is the number of errors which the code can correct. The desired bit error rate is 10^{-5} at $\frac{E_b}{N_0} = 10dB$. What modulation /coding scheme gives the best performance which meets the specifications?

$$p_{QPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$p_{8PSK} = \frac{2}{3} Q\left(\sqrt{\frac{2E_b}{N_0} \times k \times \sin^2\left(\frac{\pi}{8}\right)}\right)$, where $k = 3$ for 8-PSK modulation.