UNIVERSITY OF KWAZULU NATAL

EXAMINATIONS: JUNE 2014

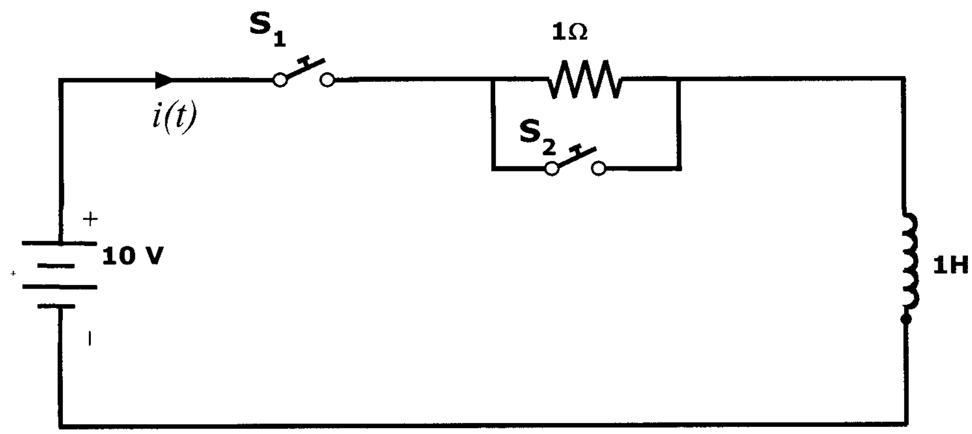
SUBJECT, COURSE AND CODE: ENEL4MA H1, ELECTRICAL MACHINES 3

DURATION: 2hrs **TOTAL MARKS: 100**

EXAMINERS:	**Mr G Diana (Internal)** **Prof WA Cronje (External)**
INSTRUCTIONS:	**Answer 4 questions only.** **Tables of standard Laplace Transforms and trigonometric identities are attached** **This paper consists of 5 pages only**

QUESTION: 1

a) A circuit consisting of a 10V dc battery in series with a 1Ω resistor and $1H$ inductor is shown together with switches S_1 and S_2 both of which are initially open.



The switches are then opened in the following sequence;

1. At t=0 switch S_1 is closed and S_2 left open until such time as current $i(t)$ reaches steady state.
2. As soon as $i(t)$ reaches steady state in 1 above switch S_2 is then also closed for a duration of 1 second.
3. After being closed for a duration of 1 second as described in 2 above switch S_2 is then opened and the system allowed to settle and reach steady-state.

For the switching sequence described in 1-3 above determine $i(t)$ fro each condition and neatly and accurately draw a diagram for the time response for $i(t)$ from t=0 until steady state is reached after 3 above.

[15 marks]

b) A separately excited dc machine has a constant field current and drives a mechanical load consisting of inertia and friction and is subject to an external mechanical load applied to the shaft.

1. Draw a block diagram which represents the internal dynamic structure of the machine with the armature voltage V_a and mechanical load T_L as inputs and the mechanical speed ω_r as output.
2. Use the block diagram derived in 1 above and perform a block diagram reduction to derive the block diagram between the input mechanical torque T_L and the output mechanical speed ω_r.
3. For the system derived in 2 above determine the steady state error.

[10 marks]

[25 marks]

QUESTION: 2

A separately excited dc motor has the following data;

$$R_a = 0.6\Omega\,; \quad L_a = 0.012H\,; \quad R_f = 240\Omega\,; \quad L_f = 120H$$

A load test was conducted during which it was found that the motor developed an electromagnetic torque of 45Nm at an armature current of 50A and a field current of 0.5A. The combined inertia of the motor and its load is 1.2 kgm²rad⁻¹. The mechanical load applied to the shaft is proportional to speed and defined as $T_L = 0.35\omega_r$ where ω_r = rad/sec.

The motor is started with 3.4Ω resistance in series with the armature to limit the starting current. A constant dc voltage of 240V is first applied to the field, and once the field current has settled and reached steady state the same voltage is applied to the armature circuit, including the 3.4Ω resistor.

Derive and express in a closed form solution as a function of time;

1. The rise of field current.
2. The rise of armature current and speed when 240V after the field current has settled and reached steady state and 240V applied to the armature circuit.

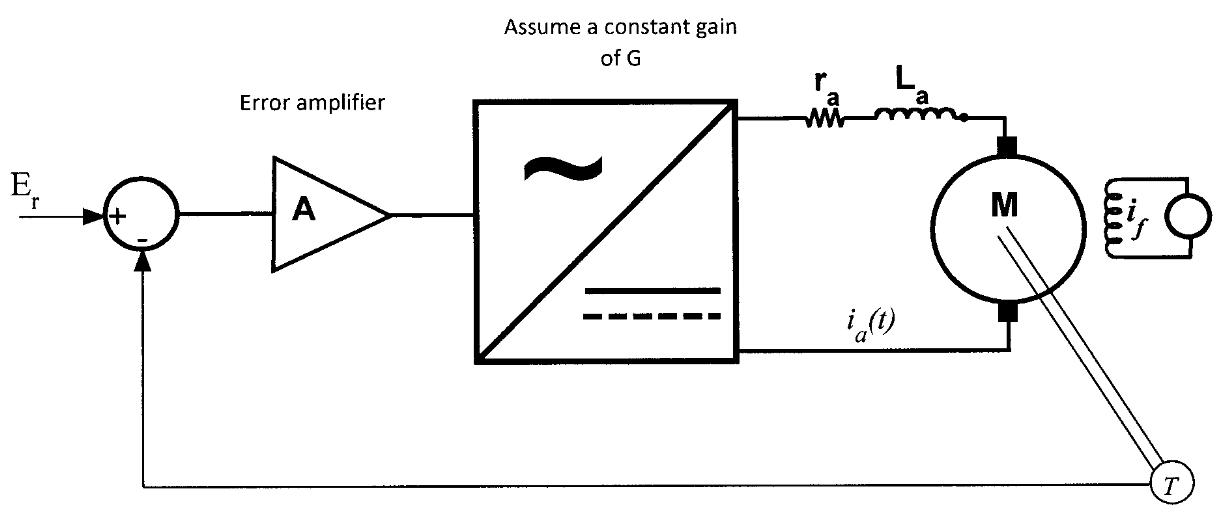
TOTAL [25 marks]

QUESTION: 3

a) For the speed control system shown below, derive the following;

1. The systems natural frequency.
2. The damping factor (α) and damping ratio (ξ).
3. An expression for the speed regulation in terms of E_r.

[15 marks]



b) If the motor has the following parameters
$R_a = 1.7\Omega\,;\; L_a = 0.016H\,;\quad K_m = 1.21\,Nm\ per\ A\ or\ V\ per\ rs^{-1}\,;\; \mathrm{J} = 0.34\ \mathrm{kgm}^{-2;}\,Kt = 0.1V/rpm,$
Calculate the combined error amplifier and Thyristor bridge gain required fo a closed loop natural frequency of 65 rad/sec. Comment on the loop damping.

[10 marks]

TOTAL [25 marks]

QUESTION: 4

a) Explain the benefits of the 2-axis theory for the simulation and analysis of electrical machines.

[5 marks]

b) The 2-axis equations which describe a symmetric squirrel cage induction machine in the stationary frame of reference are shown below. Derive the steady state equivalent circuit model clearly stating all assumption's made.

$$\begin{aligned} v_{ds} &= R_1 i_{ds} + p\lambda_{ds} - p\theta\lambda_{qs} \\ v_{qs} &= R_1 i_{qs} + p\lambda_{qs} + p\theta\lambda_{ds} \\ 0 &= R_2 i_{dr} + p\lambda_{dr} - p\beta\lambda_{qr} \\ 0 &= R_2 i_{qr} + p\lambda_{qr} + p\beta\lambda_{dr} \end{aligned}$$

Where

$$\begin{bmatrix} \lambda_{ds} \\ \lambda_{dr} \\ \lambda_{qs} \\ \lambda_{qr} \end{bmatrix} = \begin{bmatrix} L_{11} & L_m & 0 & 0 \\ L_m & L_{11} & 0 & 0 \\ 0 & 0 & L_{22} & L_m \\ 0 & 0 & L_m & L_{22} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qs} \\ i_{qr} \end{bmatrix}$$

[15 marks]

c) For a balanced three phase set of voltages $v_a(t) = VSin(\omega t + \alpha - 120); v_b(t) = VSin(\omega t + \alpha);\ v_c(t) = VSin(\omega t + \alpha - 120);$ find V_d and V_q

[5 marks]

Total [25 marks]

QUESTION: 5

a) The d-axis equivalent circuit of the synchronous machine may be represented as:

$$\lambda_d = L_{md} i_f + (L_{md} + \ell_a) i_f \qquad [1]$$

$$e_f = r_f i_f + (L_{md} + \ell_a) p i_f + L_{md} p i_d \qquad [2]$$

In many problems the current in the field winding may be eliminated to yield a single operational impedance defined as

$$\lambda_d = X_d(p) id_f + G(p) e_f$$

Show that by substituting for i_f from [2] into [1] above that

$$X_d(p) = (L_{md} + \ell_a)\frac{(1 + T'_d p)}{(1 + T'_{do} p)}) = L_d \frac{(1 + T'_d p)}{(1 + T'_{do} p)}$$

$$G(p) = \frac{L_{md}}{r_f} \frac{1}{(1 + T'_{do} p)})$$

Where

$$T'_{do} = \frac{L_{md} + L_f}{r_f} \ and\ T'_d = \frac{1}{r_f}\left(L_f + \frac{L_{md} + L_q}{L_{md} L_q}\right) = \frac{1}{r_f}\left(L_f + \frac{1}{\frac{1}{L_{md}} + \frac{1}{L_q}}\right)$$

[15 marks]

b) Show that $[P_\theta] * [\dot{P_\theta}]^{-1} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

[5 marks]

Total [25 marks]

$$[P_\theta] = \frac{2}{3}\begin{bmatrix} Cos(\theta) & Cos(\theta-120) & Cos(\theta+120) \\ Sin(\theta) & Sin(\theta-120) & Sin(\theta+120) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$[P_\theta]^{-1} = \begin{bmatrix} Cos(\theta) & Sin(\theta) & 1 \\ Cos(\theta-120) & Sin(\theta-120) & 1 \\ Cos(\theta+120) & Sin(\theta+120) & 1 \end{bmatrix}$$

Laplace transform table

Function name	Time domain function $f(t)$	Laplace transform $F(s) = L\{f(t)\}$
Constant	1	$\frac{1}{s}$
Linear	t	$\frac{1}{s^2}$
Power	t^n	$\frac{n!}{s^{n+1}}$
Exponent	e^{at}	$\frac{1}{s-a}$
Sine	$\sin at$	$\frac{a}{s^2+a^2}$
Cosine	$\cos at$	$\frac{s}{s^2+a^2}$
Growing sine	$t \sin at$	$\frac{2as}{(s^2+a^2)^2}$
Growing cosine	$t \cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
Decaying sine	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
Decaying cosine	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\sin(\theta \pm 120) = -\frac{1}{2}\sin\theta \pm \frac{\sqrt{3}}{2}\cos\theta \qquad \text{A-1}$$

$$\cos(\theta \pm 120) = -\frac{1}{2}\cos\theta \mp \frac{\sqrt{3}}{2}\sin\theta \qquad \text{A-2}$$

$$\sin^2(\theta \pm 120) = \frac{1}{4}\sin^2\theta + \frac{3}{4}\cos^2\theta \mp \frac{\sqrt{3}}{2}\sin\theta\cos\theta = \frac{1}{2} + \frac{1}{4}\cos 2\theta \mp \frac{\sqrt{3}}{4}\sin 2\theta \qquad \text{A-3}$$

$$\cos^2(\theta \pm 120) = \frac{1}{4}\cos^2\theta + \frac{3}{4}\sin^2\theta \pm \frac{\sqrt{3}}{2}\sin\theta\cos\theta = \frac{1}{2} - \frac{1}{4}\cos 2\theta \pm \frac{\sqrt{3}}{4}\sin 2\theta \qquad \text{A-4}$$

$$\sin\theta\sin(\theta \pm 120) = -\frac{1}{2}\sin^2\theta \pm \frac{\sqrt{3}}{2}\sin\theta\cos\theta = -\frac{1}{4} + \frac{1}{4}\cos 2\theta \pm \frac{\sqrt{3}}{4}\sin 2\theta \qquad \text{A-5}$$

$$\cos\theta\cos(\theta \pm 120) = -\frac{1}{2}\cos^2\theta \mp \frac{\sqrt{3}}{2}\sin\theta\cos\theta = -\frac{1}{4} - \frac{1}{4}\cos 2\theta \mp \frac{\sqrt{3}}{4}\sin 2\theta \qquad \text{A-6}$$

$$\sin\theta\cos(\theta \pm 120) = -\frac{1}{2}\sin\theta\cos\theta \mp \frac{\sqrt{3}}{2}\sin^2\theta = \mp\frac{\sqrt{3}}{4} - \frac{1}{4}\sin 2\theta \pm \frac{\sqrt{3}}{4}\cos 2\theta \qquad \text{A-7}$$

$$\cos\theta\sin(\theta \pm 120) = -\frac{1}{2}\sin\theta\cos\theta \pm \frac{\sqrt{3}}{2}\cos^2\theta = \pm\frac{\sqrt{3}}{4} - \frac{1}{4}\sin 2\theta \pm \frac{\sqrt{3}}{4}\cos 2\theta \qquad \text{A-8}$$

$$\sin(\theta \pm 120)\cos(\theta \mp 120) = \sin\theta\cos\theta \mp \frac{\sqrt{3}}{4} = \frac{1}{2}\sin 2\theta \pm \frac{\sqrt{3}}{4} \qquad \text{A-9}$$

$$\sin(\theta + 120)\sin(\theta - 120) = \frac{1}{4}\sin^2\theta - \frac{3}{4}\cos^2\theta = -\frac{1}{4} - \frac{1}{2}\cos 2\theta \qquad \text{A-10}$$

$$\cos(\theta + 120)\cos(\theta - 120) = \frac{1}{4}\cos^2\theta - \frac{3}{4}\sin^2\theta = -\frac{1}{4} + \frac{1}{2}\cos 2\theta \qquad \text{A-11}$$

$$\sin(2\theta \pm 120) = -\frac{1}{2}\sin 2\theta \pm \frac{\sqrt{3}}{2}\cos 2\theta \qquad \text{A-12}$$

$$\sin(\theta \pm 120)\cos(\theta \pm 120) = -\frac{1}{2}\sin\theta\cos\theta \mp \frac{\sqrt{3}}{4}\cos^2\theta \pm \frac{\sqrt{3}}{4}\sin^2\theta = -\frac{1}{4}\sin 2\theta \mp \frac{\sqrt{3}}{4}\cos 2\theta \qquad \text{A-13}$$

$$\sin\theta + \sin(\theta - 120) + \sin(\theta + 120) = 0 \qquad \text{A-14}$$

$$\cos\theta + \cos(\theta - 120) + \cos(\theta + 120) = 0 \qquad \text{A-15}$$

$$\sin^2\theta + \sin^2(\theta - 120) + \sin^2(\theta + 120) = \frac{3}{2} \qquad \text{A-16}$$

$$\cos^2\theta + \cos^2(\theta - 120) + \cos^2(\theta + 120) = \frac{3}{2} \qquad \text{A-17}$$

$$\sin\theta\cos\theta + \sin(\theta - 120)\cos(\theta - 120) + \sin(\theta + 120)\cos(\theta + 120)) = 0 \qquad \text{A-18}$$