University of KwaZulu-Natal

ENEL3CSH2 Control Systems 1: **November 2016 Main Examinations**

Time Allowed: 2 hours **Maximum points: 100.**

Provided: graph paper, formula sheet, computer sheet

Examiner: Prof. J. T. Agee

Instructions:

This paper has seven sections. Sections 1-6 consist of multiple choice questions. For these questions, select the letter you consider the most appropriate solution for each question and enter your answer on the computer sheet provided. The objective questions shall be marked by a computer programme. Make sure you follow the instructions detailed on the computer sheet.

Answer all questions.

Section one [12 marks]

Consider Figure 1, where R(s) is the input, N(s) is a disturbance, $G_d(s)$ is the transfer function of the feedforward controller, and Y(s) is the output of the system. Answer questions 1-4., based on Figure 1:

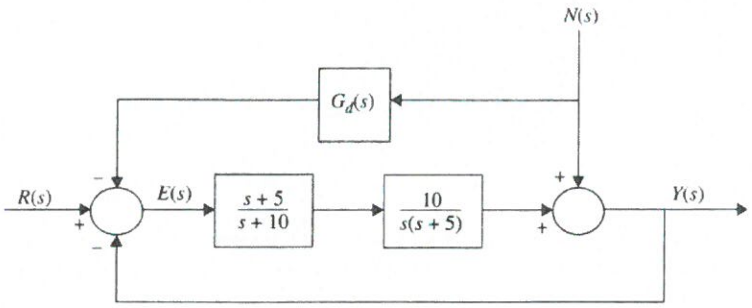

Figure 1

1. Obtain the transfer function $\left.\dfrac{Y(s)}{R(s)}\right|_{N=0}$ **[3]**

(a) $\dfrac{10s}{s^2+10s+10}$ (b) $\dfrac{s}{s^2+10s+10}$ (c) $\dfrac{10}{s^2+10s+10}$ (d) $\dfrac{1}{s^2+10s+10}$

2. Obtain the transfer function $\left.\frac{Y(s)}{N(s)}\right|_{R=0}$ without the feedforward controller gain $G_d(s)$ **[3]**

(a) $\frac{1}{s^2+10s+10}$ (b) $\frac{10}{s^2+10s+10}$ (c) $\frac{s^2+10s}{s^2+10s+10}$ (d) $\frac{s+10}{s^2+10s+10}$

3. Obtain the transfer function $\left.\frac{Y(s)}{N(s)}\right|_{R=0}$ with the feedforward controller gain, $G_d(s)$, connected **[3]**

(a) $\frac{1-10G_d(s)}{s^2+10s+10}$ (b) $\frac{10-10G_d(s)}{s^2+10s+10}$ (c) $\frac{s^2+10s-10G_d(s)}{s^2+10s+10}$

(d) $\frac{s+10-10G_d(s)}{s^2+10s+10}$

4. Determine the expression for $G_d(s)$, required to eliminate the effect of the disturbance N(s) on the output Y(s) **[3]**

(a) $0.1s^2+s$ **(b)** $10s^2+s$ **(c)** $0.1s+1$ **(d)** s^2+s

Section two [10 marks]

The open loop system with the transfer function $G(s)=\frac{25}{s(s+2)}$***, is used in unity feedback; answer questions 5-9:***

5. Natural frequency of the resulting system **[2]**

(a) 2rad/s (b) 25 rad/s (c) 5 rad/s (d) 2.5rad/s

6. The damping factor of the system **[2]**

(a) 1 (b) 0 (c) 0.4 (d) 0.2

7. The settling time **[2]**

(a) 1 s (b) 2 s (c) 4 s (d) 0.5 s

8. The percentage overshoot **[2]**

(a) 52.66 (b) 42.66 (c) 62.66 (d) 48

9. The rise time of the system **[2]**

(a) 0.341s (b) 0.241s (c) 0.141s (d) 0.421s

Section three [8 marks]

A parabolic input is applied to the system in Figure 2. Questions 10-13 are to be answered based on Figure 2:

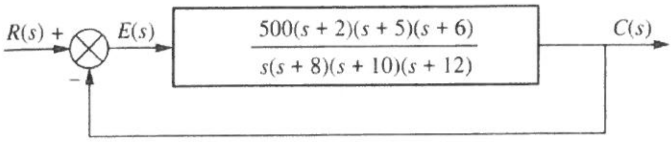

Figure 2

10. Obtain the positional error constant K_P **[2]**

(a) 31.25 (b) 0 (c) ∞ (d) 5.208

11. Obtain the steady-state positional error **[2]**

(a) 0.031 (b) ∞ (c) 0 (d) 0.1611

12. Evaluate the velocity error constant K_V **[2]**

(a) 31.25 (b) 0 (c) ∞ (d) 5.208

13. Evaluate the steady-state velocity error **[2]**

(a) 0.032 (b) ∞ (c) 0 (d) 0.1920

Section four [3 marks]

The characteristic equation of a system is given by $Q(s) = s^3 + 2s^2 + s + 2 = 0$***. Answer question 14:***

14. Determine the frequency of oscillations for the system, using the Routh-Hurwitz method **[3]**

(a) 0.1 rad/s (b) 1 rad/s (c) 1.5 rad/s (d) 2 rad/s

Section five [6 marks]

The continued fraction expansion of the characteristic equation $Q(s) = s^3 + 6s^2 + 12s + 8 = 0$ ***yields***

$\frac{Q_1(s)}{Q_2(s)} = h_1 s + \frac{1}{h_2 s + \frac{1}{h_3 s}}$***, answer questions 15-17:***

15. What is the value of h_1? **[2]** (a) 6 (b) 1/6 (c)=3 (d) 2

16. What is the value of h_2?   **[2]**   (a) 16 (b) 9   (c) 9/16   (d) 16/19

17. What is the value of h_3?   **[2]**   (a)=4/3 (b)=3/4 (c) 3   (d) 4

Section six [4 marks]

Consider the Nyquist polar plot shown in Figure 3. Answer Question 18-19:

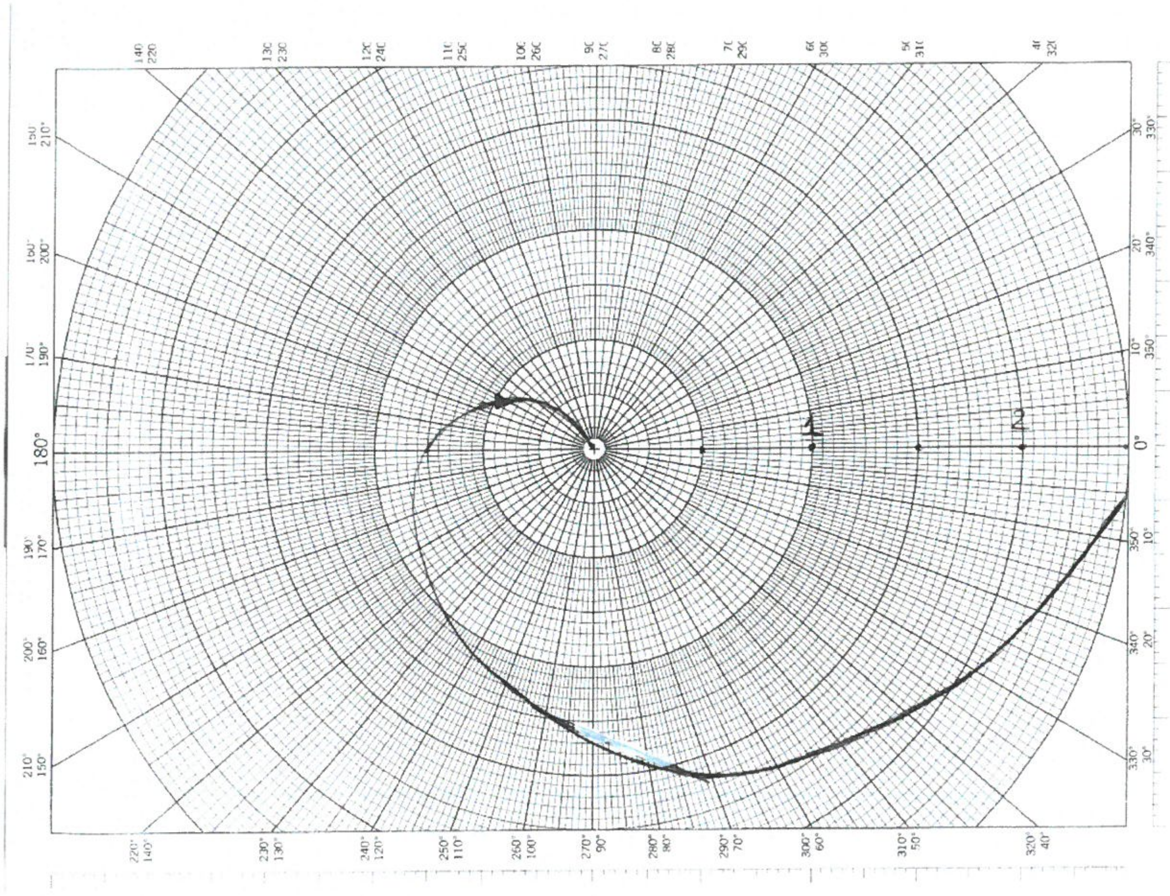

Figure 3: Nyquist Polar plot

(18) Determine the gain margin **[2]**

(a) 133 (b) 0.75 (c) 1.333 (d) 0.0075

(19) Determine the phase margin **[2]**

(a) 47^0 (b) 75^0 (c) 133^0 (d) 105^0

Section Seven [57 marks]

20 The system of equations describing the dynamic behaviour of a certain control system is given by:

$$x_1 = r(t) - 10x_2$$

$$x_2 = 6x_1 + \frac{dy}{dt}$$

$$\frac{dx_3}{dt} + 3x_3 = \frac{dx_2}{dt}$$

$$y = 3x_3 + \int x_1 dt$$

(a) Draw the signal flow graph representing the given system of equations **[10]**

(b) Using the Mason's gain rule, obtain the transfer function Y(s)/R(s). **[10]**

21. Convert the system in Figure 4 into an equivalent signal flow graph **[11]**

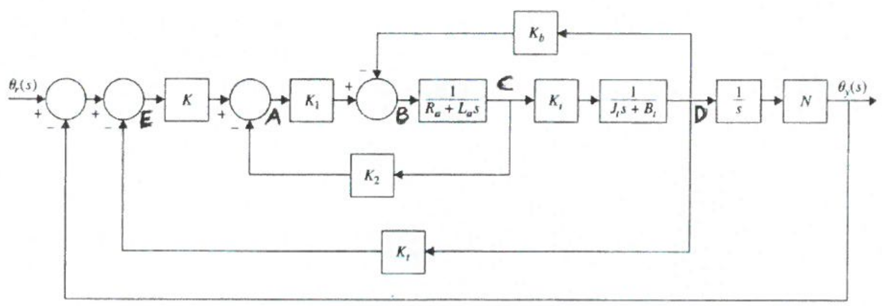

Figure 4: A block diagram

22. The open-loop transfer function $G(s) = 50\frac{0.001N}{s(s+5)(0.05s+0.5)}$ is used in unity feedback.

(a) Draw a detailed roots-locus plot of the system if **N** is the gain of the system **[10]**

(b) Using the roots-locus plot, show that N =128, when the closed-loop system has a damping factor of $\xi = 0.5$ **[4]**

(c) Evaluate the steady state velocity error for the system when $\xi = 0.5$ **[4]**

(d) Obtain the transfer function for a series lag compensator designed for the system, such that the steady-state error is reduced by a factor of 10, without changing the damping factor of the system. Assume that the compensator has a pole located at **s = -0.015rad/s [8]**

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Modeling

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} \quad (2.97); \qquad \frac{V_o(s)}{V_i(s)} = \frac{Z_1(s)+Z_2(s)}{Z_1(s)} \quad (2.104)$$

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} \quad (2.133); \qquad \frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} \quad (2.135)$$

$$\left(\frac{\text{Number of teeth of gear on } \textit{destination} \text{ shaft}}{\text{Number of teeth of gear on } \textit{source} \text{ shaft}}\right)^2 \quad \text{(see after 2.138)}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]} \quad (2.153)$$

$$\frac{K_t}{R_a} = \frac{T_{\text{stall}}}{e_a} \quad (2.162); \qquad K_b = \frac{e_a}{\omega_{\text{no-load}}} \quad (2.163)$$

$$T(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \quad (3.73)$$

Time Response

$$T_r = \frac{2.2}{a} \quad (4.9); \qquad T_s = \frac{4}{a} \quad (4.10)$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4.22)$$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100 \quad (4.38)$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} \quad (4.39)$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \quad (4.34); \qquad T_s = \frac{4}{\zeta\omega_n} \quad (4.42)$$

Steady-State Error

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1+\lim_{s\to 0} G(s)} \quad (7.30); \qquad K_p = \lim_{s\to 0} G(s) \quad (7.33)$$

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} \quad (7.31); \qquad K_v = \lim_{s\to 0} sG(s) \quad (7.34)$$

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s\to 0} s^2G(s)} \quad (7.32); \qquad K_a = \lim_{s\to 0} s^2G(s) \quad (7.35)$$

Root Locus

$$\angle KG(s)H(s) = -1 = 1\angle(2k+1)180^\circ \quad (8.13)$$

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{ finite poles} - \#\text{ finite zeros}} \quad (8.27)$$

$$\theta_a = \frac{(2k+1)\pi}{\#\text{ finite poles} - \#\text{ finite zeros}} \quad (8.28)$$

$$\theta = \sum \text{finite zero angles} - \sum \text{finite pole angles}$$

$$K = \frac{1}{|G(s)H(s)|} = \frac{1}{M} = \frac{\prod \text{finite pole lengths}}{\prod \text{finite zero lengths}} \quad (8.51)$$

Frequency Response

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (10.52); \qquad \omega_p = \omega_n\sqrt{1-2\zeta^2} \quad (10.53)$$

$$\omega_{BW} = \omega_n\sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \quad (10.54)$$

$$\Phi_M = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}} \quad (10.73)$$

$$\phi_{\max} = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}} = \sin^{-1}\frac{1-\beta}{1+\beta} \quad (11.11)$$

$$\omega_{\max} = \frac{1}{T\sqrt{\beta}} \quad (11.9); \qquad |G_c(j\omega_{\max})| = \frac{1}{\sqrt{\beta}} \quad (11.12)$$

State Space

$$\mathbf{C_M} = [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B} \quad \cdots \quad \mathbf{A}^{n-1}\mathbf{B}] \quad (12.26)$$

$$\dot{\mathbf{x}} = (\mathbf{A}-\mathbf{BK})\mathbf{x} + \mathbf{B}r; \quad y = \mathbf{Cx} \quad (12.3); \qquad \mathbf{O_M} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix} \quad (12.79)$$

$$\dot{\mathbf{e}}_x = (\mathbf{A}-\mathbf{LC})\mathbf{e}_x; \quad y - \hat{y} = \mathbf{C}\mathbf{e}_x \quad (12.64)$$

Digital Control

$$e^*(\infty) = \lim_{z\to 1}(1-z^{-1})E(z) \quad (13.66)$$

$$K_p = \lim_{z\to 1} G(z) \quad (13.70); \qquad K_v = \frac{1}{T}\lim_{z\to 1}(z-1)G(z) \quad (13.73)$$

$$K_a = \frac{1}{T^2}\lim_{z\to 1}(z-1)^2G(z) \quad (13.75)$$

$$T_r = \frac{1}{\omega_n}\left\{1.76\xi^3 - 0.417\xi^2 + 1.039\varsigma + 1\right\}$$

$$s_1, s_2 = -\xi\omega_n + \omega_n\sqrt{1-\xi^2}$$

Lag control: $G_C(s) = K_C\left(\frac{s+1/T}{s+1/\alpha T}\right); \alpha > 1, K_C = \frac{1/\alpha T}{1/T} = \frac{1}{\alpha}$

Lead Control:

$$G_C(s) = \frac{1}{\beta}\left(\frac{s+1/T}{s+1/\beta T}\right); \beta < 1; \qquad G_C(j\omega_{\max}) = \frac{1}{\sqrt{\beta}}$$

$$\omega_{\max} = \frac{1}{T\sqrt{\beta}}; \phi_{\max} = \sin^{-1}\left\{\frac{1-\beta}{1+\beta}\right\}, \beta = \frac{1-\sin\phi_{\max}}{1+\sin\phi_{\max}}$$

	K_c	T_i	T_d
P	K_0		
PI	$0.9K_0$	$3.3\tau_{dead}$	
PID	$1.2K_0$	$2\tau_{dead}$	$0.5\tau_{dead}$