University of KwaZulu-Natal
School of Engineering

Superconductivity I (DNE4SC1)

Internal Examiner: Dr Leigh. Jarvis — Date: October 2016
External Examiner: Professor W.J. Perold — Time: 2 Hours
Total Marks: 100

Instructions: Answers ALL questions

NOTE APPENDIX A & B ATTACHED

Data:

$Y_1Ba_2Cu_3O_{7-\delta}(Tc) = 92.5K$ $\quad eV = 1.6022\ x10^{-19}\ J$ $\quad m_e = 9.1095\ x10^{-31}\ kg$

$\bar{e} = 1.6022\ x10^{-19}\ C$ $\quad h = 6.6262\ x10^{-34}\ J.s$ $\quad \hbar = \frac{h}{2\pi} = 1.0546\ x10^{-34}\ J.s$

$\mu_0 = 4\pi\ x10^{-7}\ H/m$ $\quad \varepsilon_0 = 8.8542\ x10^{-12}\ F/m$ $\quad \Phi_0 = \frac{h}{2e} = 2.0679\ x10^{-15}\ T.m^2$

$k_B = 1.3807\ x10^{-23}\ J/K$

Question 1 – *Superconductor Applications* [15]

(a) Explain the technological advancement from first to second generation HTS wire/tape, with a focus on materials. Then briefly discuss second generation tape.

(8)

(b) Why do some second generation tape include ‘nanodots’ in the tape?

(3)

(c) Current surge faults on the grid are mitigated by Eskom by use of series core reactors (SCRs). What superconductor technology could replace an SCR and explain how it works?

(4)

Question 2 – *Superconductor vs. Perfect Conductor* [6]

(a) Give two differences how a ‘perfect conductor’ differs from a superconductor?

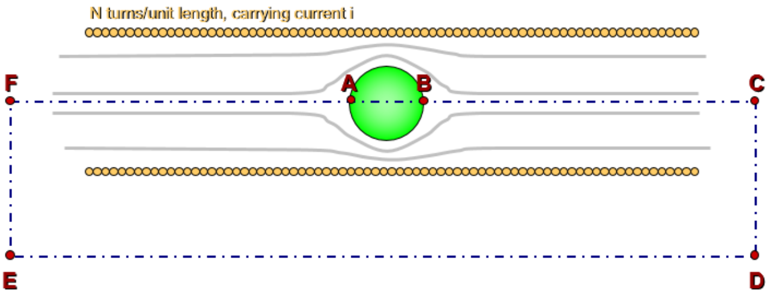
(3)

(b) What two effects does one observe when a magnet levitates above a superconductor?

(3)

Question 3 – *Superconductor in a Solenoid* [10]

The below schematic is a longitudinal cross-section of a solenoid with a solid spherical superconductor placed in the centre of the solenoid. The solenoid has *N* turns per unit length and is carry current, *i*.



$$\oint \underline{H} \cdot \underline{dl} = \int_{AB} \underline{H}_i \cdot \underline{dl} + \int_{BCDEF} \underline{H}_e \cdot \underline{dl}$$

Show by starting with the integral expression above, how the internal field, H_i, within a superconductor can be 'greater' than the applied field, H_a. H_e, is the external field when the superconductor is in the solenoid.

(10)

Question 4 – *Phenomenological Theory* [14]

(a) Prove the theoretical relationship that relates electric field to the rate of current density in a superconductor? Name this relationship.

(6)

(b) Using your answer in part (a) derive the following expression for the London penetration depth, $\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$.

(8)

Question 5 – *Charge carriers* and HTS [10]

(a) What is the name given to the Boson charge carrier of a superconductor and what is it?

(2)

(b) Discuss the isotope effect.

(4)

(c) Why could one refer to High Temperature Superconductors as two dimensional superconductors?

(4)

Question 6 – Type II Superconductors [15]

(a) Draw the magnetisation curve, M(Ha), for a type-I and type-II superconductor.

(5)

(b) Prove from an energy perceptive, why magnetic cores will spontaneously form in a type-II superconductor.

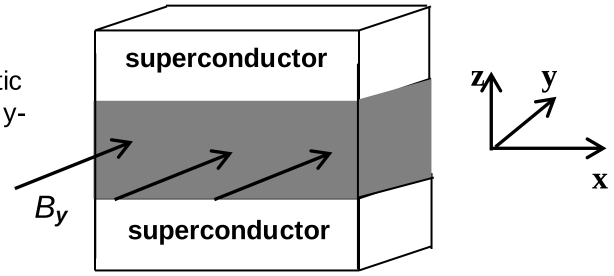
(10)

Question 7 – *Josephson Current vs. Magnetic field* [10]

For the Josephson junction shown schematically below, derive the following expression for the modulation of the current *I* in the junction,

$$I = L_x D_z J_0 \frac{\sin \pi\Phi / \Phi_0}{\pi\Phi / \Phi_0} \sin \gamma_0 \, .$$

B_y is a uniform magnetic field applied along the y-axis



(10)

Question 8 – Superconductor Electronics [20]

Refer to Appendix A for parts of this question

(a) Explain and draw the VI characteristics of a NIS junction (8)

(b) Draw a circuit model of a DC-SQUID (6)

(b) (i) Calculate the modulation parameter, β_L of a DC SQUID from the data in Appendix A. (3)

(ii) Explain how you could calculate the Inductance of the SQUID from (i)? (3)

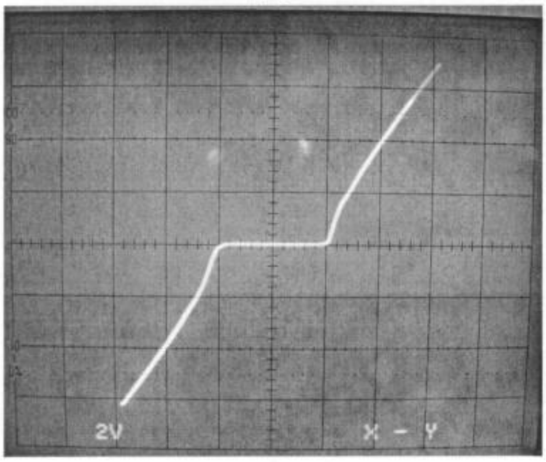
TOTAL MARKS = 100

Appendix A

Characteristic measurements of a HTC SQUID and one of its junctions.

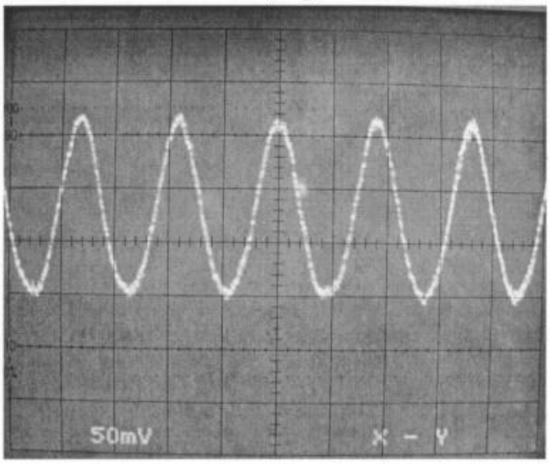
$2I_c$:	90 μA	**$R_n/2$:**	2.4 Ω
Mod. Depth:	16 μV	**Int. Coil Coupling:**	90 $\mu A/\Phi_0$
		Ext. Coil Coupling:	59 $\mu A/\Phi_0$

V-I



V: 200 μV/div., H: 100 μA/div.

V-Φ



V: 5 μV/div., H: 50 μA/div.

Notes: Flux bias adjusted to maximize positive I_c in *V-I* characteristic.

All measurements at approx. 75K

Appendix B – Formulae Sheet

This formulae sheet is not considered a complete

$\underline{J} = ne\underline{v}$

$\dot{\underline{B}} = -curl\,\underline{E}$

$E = \frac{1}{2}\mu_o H_a^2$

$|\Psi|^2 = \frac{-\alpha}{\beta}$

$\xi^2 = \frac{\hbar^2}{2m^*|\alpha|}$

$\alpha\Psi + \beta|\Psi|^2\Psi + \frac{1}{2m^*}(i\hbar\nabla + e^*\underline{A})^2\Psi = 0$

$\gamma = \varphi_2 - \varphi_1 - \frac{2e}{\hbar}\int_1^2 \overline{A}\cdot\overline{dl}$

$curl\,\underline{B} = \mu_o \underline{J}$

$div\,\dot{\underline{B}} = 0$

$B_y(x) = -\frac{\partial A_z}{\partial x}$

$J(x) = J_0 \sin\gamma(x)$

$E_z = -\frac{\partial A_z}{\partial t}$

$curl\,\underline{H} = \underline{J} + \dot{\underline{D}}$

$curl\,curl\,\dot{\underline{B}} = grad\ div\,\dot{\underline{B}} - \nabla^2\dot{\underline{B}}$

$\omega_0 = \frac{2e}{\hbar}V_0$

$\beta_L = \frac{4I_c R_n}{\pi\Delta V} - 1$

$\beta_L = \frac{2I_c L}{\Phi_0}$

$\beta_c = \frac{2\pi}{\Phi_0}R_n^2 I_c C$

Normal core energy increase:

$\pi\xi^2\,\frac{1}{2}\mu_o H_c^2$

Local vortex energy decrease:

$\pi\lambda^2\,\frac{1}{2}\mu_o H_a^2$